

# **Analytical approaches for large-scale structures in some modified-gravity scenarios**

P. Valageas

IPhT - CEA Saclay

- Modified gravity models
- Perturbative approach (SPT)
- Spherical collapse
- Matter power spectrum (SPT+halo model)
- Density probability distribution
- Conclusion

Accelerated expansion of the Universe →

- dark energy

- modified gravity

Most of the models involve one or more scalar fields, which experience self-interactions and may also interact with matter.

→ “Fifth force” that has **not** been **seen** in local gravity experiments !

- the scalar field does not interact with baryonic matter components

- there is a mechanism to suppress the fifth force in local environments

→ “Screening” mechanisms associated with non-linearities of the system.

# **Definitions of some models**

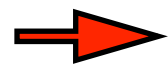
## a) f(R) models

Add a function of the Ricci scalar,  $f(R)$ ,  
to the Einstein-Hilbert action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} (R + f(R)) + \mathcal{L}_m \right]$$

$$f(R) = -2\Lambda - \frac{f_{R_0} c^2}{n} \frac{R_0^{n+1}}{R^n} \quad n = 1, \quad |f_{R_0}| \leq 10^{-5}$$

Solar System constraints



Modified Poisson equation:

$$\nabla^2 \Psi = \frac{16\pi\mathcal{G}}{3} a^2 \delta\rho - \frac{a^2}{6} \delta R$$

Constraint equation:  $\nabla^2 \delta f_R = \frac{a^2}{3} [\delta R - 8\pi\mathcal{G}\delta\rho]$

$$f_R = \frac{df}{dR} = f_{R_0} c^2 \frac{R_0^{n+1}}{R^{n+1}}$$

(quasi-static approximation)

## b) Scalar field models

### Dilaton models or symmetron models

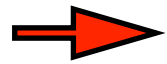
Scalar-tensor theories

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\nabla\varphi)^2 - V(\varphi) \right] + \int d^4x \sqrt{-\tilde{g}} \mathcal{L}_m(\psi_m, \tilde{g}_{\mu\nu})$$

$$\tilde{g}_{\mu\nu} = A(\varphi)^2 g_{\mu\nu}$$

Jordan-frame metric

Einstein-frame metric



Modified Poisson equation (5th force):

$$\Psi = \Psi_N + \Psi_A$$

$$\nabla^2 \Psi_N = 4\pi \mathcal{G} a^2 \delta\rho$$

$$\Psi_A = c^2 (A - \bar{A})$$

$$(A \simeq 1)$$

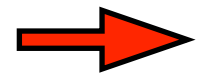
Klein-Gordon eq.:

$$\frac{c^2}{a^2} \nabla^2 \varphi = \frac{dV}{d\varphi} + \rho \frac{dA}{d\varphi}$$

(quasi-static approximation)

# **Perturbative approach**

1) Use the **quasi-static** approximation, which applies to small scales dominated by spatial gradients



Obtain a **non-linear** equation that relates the new field to the matter density

$$\mathcal{F}(\delta R, \delta\rho) = 0$$

$$\mathcal{F}(\delta\varphi, \delta\rho) = 0$$

This allows one to eventually go back to the standard LCDM formalism (i.e., we can **eliminate** the new degree of freedom).

2) Solve this equation through a perturbative **expansion over the nonlinear density** fluctuation

$$\delta\tilde{R}(\mathbf{k}) = \sum_{n=1}^{\infty} \int d\mathbf{k}_1 \dots d\mathbf{k}_n \delta_D(\mathbf{k}_1 + \dots + \mathbf{k}_n) h_n(\mathbf{k}_1, \dots, \mathbf{k}_n) \delta\tilde{\rho}(\mathbf{k}_1) \dots \delta\tilde{\rho}(\mathbf{k}_n)$$
$$\delta\tilde{\varphi}(\mathbf{k}) = \sum_{n=1}^{\infty} \int d\mathbf{k}_1 \dots d\mathbf{k}_n \delta_D(\mathbf{k}_1 + \dots + \mathbf{k}_n) h_n(\mathbf{k}_1, \dots, \mathbf{k}_n) \delta\tilde{\rho}(\mathbf{k}_1) \dots \delta\tilde{\rho}(\mathbf{k}_n)$$

3) Obtain the expression of the **full “gravitational” potential** (Newton+5th force)

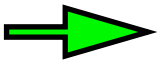
$$\tilde{\Psi}(\mathbf{k}) = \sum_{n=1}^{\infty} \int d\mathbf{k}_1 \dots d\mathbf{k}_n \delta_D(\mathbf{k}_1 + \dots + \mathbf{k}_n) H_n(\mathbf{k}_1, \dots, \mathbf{k}_n) \delta\tilde{\rho}(\mathbf{k}_1) \dots \delta\tilde{\rho}(\mathbf{k}_n)$$



## a) f(R) models

★ Constraint equation:  $\nabla^2 \delta f_R = \frac{a^2}{3} [\delta R - 8\pi\mathcal{G}\delta\rho]$   $n \geq 1: \kappa_n(a) = H^{2n-2} \frac{d^n f_R}{dR^n}(\bar{R})$

$$\left(1 - \frac{\nabla^2}{a^2 m^2}\right) \cdot \delta R = \frac{\delta\rho}{M_{\text{Pl}}^2} + \sum_{n=2}^{\infty} \frac{3H^{2-2n}\kappa_n}{a^2 n!} \nabla^2 (\delta R)^n$$

★ Modified Poisson equation:  $\nabla^2 \Psi = \frac{16\pi\mathcal{G}}{3} a^2 \delta\rho - \frac{a^2}{6} \delta R$    $\Psi(\delta\rho)$

## b) Scalar field models (Dilaton or symmetrons)

★ Klein-Gordon eq.:  $\frac{c^2}{a^2} \nabla^2 \varphi = \frac{dV}{d\varphi} + \rho \frac{dA}{d\varphi}$   $n \geq 1: \beta_n(a) = M_{\text{Pl}}^n \frac{d^n A}{d\varphi^n}(\bar{\varphi})$

$$\left(\frac{\nabla^2}{a^2} - m^2\right) \cdot \delta\varphi = \frac{\beta \delta\rho}{c^2 M_{\text{Pl}}} + \frac{\beta_2 \delta\rho}{c^2 M_{\text{Pl}}^2} \delta\varphi + \sum_{n=2}^{\infty} \left(\frac{\kappa_{n+1}}{M_{\text{Pl}}^{n-1}} + \frac{\beta_{n+1} \delta\rho}{c^2 M_{\text{Pl}}^{n+1}}\right) \frac{(\delta\varphi)^n}{n!}$$

$$n \geq 2: \kappa_n(a) = \frac{M_{\text{Pl}}^{n-2}}{c^2} \left[ \frac{d^n V}{d\varphi^n}(\bar{\varphi}) + \bar{\rho} \frac{d^n A}{d\varphi^n}(\bar{\varphi}) \right]$$

★ Modified Poisson equation (5th force):  $\Psi = \Psi_N + \Psi_A$   $\Psi_A = c^2 (A - \bar{A})$

$$\Psi_A = \sum_{n=1}^{\infty} \frac{c^2 \beta_n}{M_{\text{Pl}}^n n!} (\delta\varphi)^n$$
  $\Psi(\delta\rho)$

4) Write the equations of motion (in the single-stream approx.), with the “new gravitational potential”

Continuity eq.:  $\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1 + \delta)\mathbf{v}] = 0$

(K-mouflage models)

Euler eq.:  $\frac{\partial \mathbf{v}}{\partial \tau} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \mathcal{H}\mathbf{v} = -\nabla\Psi$

$\frac{\partial \mathbf{v}}{\partial \tau} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \left(\mathcal{H} + \frac{d \ln \bar{A}}{d\tau}\right)\mathbf{v} = -\nabla\Psi$

5th force

friction

5th force

This can be written in a more concise form as:

$$\mathcal{O}(x, x') \cdot \tilde{\psi}(x') = \sum_{n=2}^{\infty} K_n^s(x; x_1, \dots, x_n) \cdot \tilde{\psi}(x_1) \dots \tilde{\psi}(x_n)$$

2-component vector:  $\psi = \begin{pmatrix} \delta \\ -(\nabla \cdot \mathbf{v})/\dot{a} \end{pmatrix}$

time coordinate:  $\eta = \ln(a)$

$x = (\mathbf{k}, \eta, i)$

Linear part:  $\mathcal{O}(x, x') = \delta_D(\eta' - \eta) \delta_D(\mathbf{k}' - \mathbf{k}) \begin{pmatrix} \frac{\partial}{\partial \eta} & -1 \\ -\frac{3}{2}\Omega_m(1 + \epsilon_1) & \frac{\partial}{\partial \eta} + \frac{1-3w_\varphi^{\text{eff}}\Omega_\varphi^{\text{eff}}}{2} + \epsilon_2 \end{pmatrix}$

modified-gravity impact at linear order

Equal-time kernels:

$$K_n^s = \delta_D(\eta_1 - \eta) \dots \delta_D(\eta_n - \eta) \delta_D(\mathbf{k}_1 + \dots + \mathbf{k}_n) \gamma_{i_1, \dots, i_n}^s(\mathbf{k}_1, \dots, \mathbf{k}_n; \eta)$$

new time and scale dependence

## 5) Linear theory

★  $f(R)$  and dilaton models:

$$\frac{\partial^2 D}{\partial \eta^2} + \frac{1 - 3w\Omega_{\text{de}}}{2} \frac{\partial D}{\partial \eta} - \frac{3}{2} \Omega_m [1 + \epsilon(k, \eta)] D = 0$$

scale-dependence of the linear modes  
(scale and time dependent  
effective Newton constant)

★ K-mouflage models:

$$\frac{d^2 D}{d\eta^2} + \left( \frac{1 - 3w_\varphi^{\text{eff}} \Omega_\varphi^{\text{eff}}}{2} + \epsilon_2 \right) \frac{dD}{d\eta} - \frac{3}{2} \Omega_m (1 + \epsilon_1) D = 0$$

scale-independent modified  
linear growing mode

★  $f(R) : \epsilon(k, \eta) = \frac{k^2}{3(a^2 m^2 + k^2)}$

scalar field models:

$$\epsilon(k, \eta) = \frac{2\beta^2 k^2}{a^2 m^2 + k^2}$$

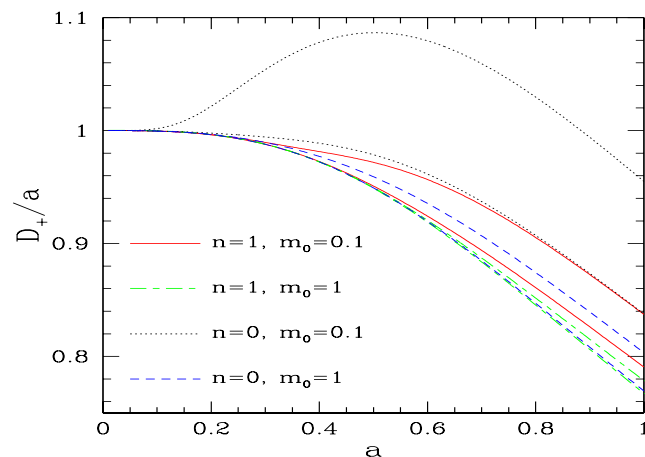


FIG. 1: Linear growing mode  $D_+(k, t)$  normalized to the scale factor  $a(t)$  for four  $(n, m_0)$  models. In each case we show the results for wavenumbers  $k = 1h\text{Mpc}^{-1}$  (lower curve) and  $k = 5h\text{Mpc}^{-1}$  (upper curve), as a function of  $a(t)$ . These two scales are in the non-linear regime and have only been chosen to exemplify the type of effects obtained in modified gravity.

Linear growing mode as  
a function of time

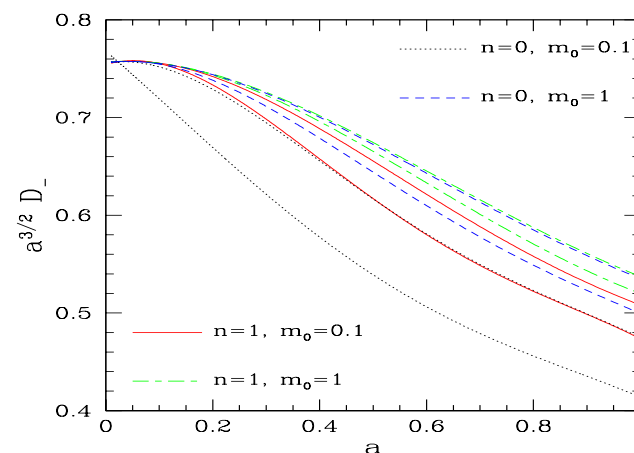


FIG. 2: Linear decaying mode  $D_-(k, t)$  normalized to  $a(t)^{-3/2}$  for four  $(n, m_0)$  models. In each case we show the results for wavenumbers  $k = 1h\text{Mpc}^{-1}$  (upper curve) and  $5h\text{Mpc}^{-1}$  (lower curve), as a function of  $a(t)$ . These two scales are in the non-linear regime and have only been chosen to exemplify the type of effects obtained in modified gravity.

Linear decaying mode as  
a function of time

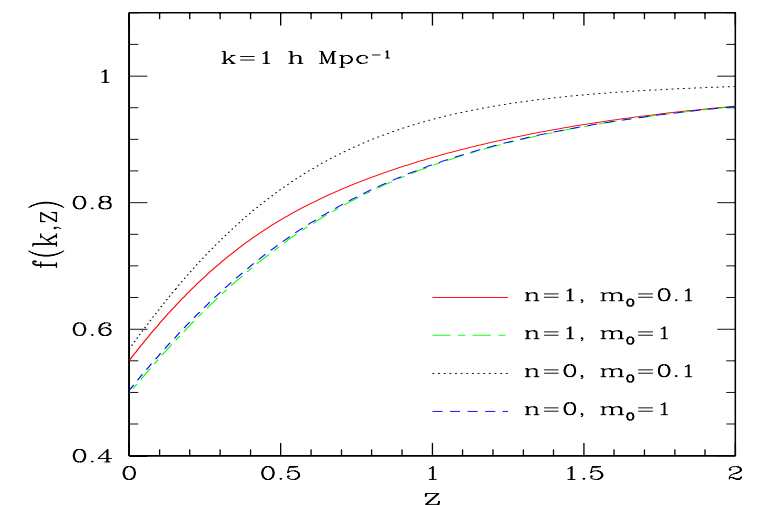


FIG. 5: Linear growth rate  $f(k, z) = \partial \ln D_+ / \partial \ln a$  for wavenumber  $k = 1h\text{Mpc}^{-1}$ , for four  $(n, m_0)$  models.

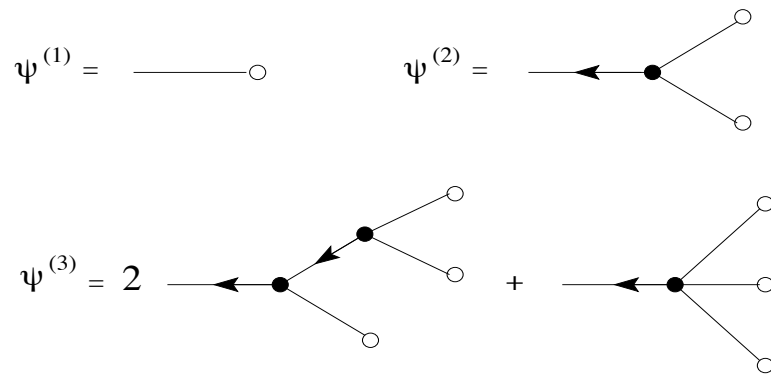
Linear growth rate as  
a function of time

## 6) One-loop power spectrum

As in the LCDM case, we can write the solution of the equation of motion as a **perturbative expansion** over powers of the linear growing mode:

$$\tilde{\psi}(x) = \sum_{n=1}^{\infty} \tilde{\psi}^{(n)}(x), \quad \text{with } \tilde{\psi}^{(n)} \propto (\tilde{\psi}_L)^n$$

Diagrams:



- white circles: linear solution
- black dots: vertices
- lines with an arrow: retarded propagator

new cubic vertex

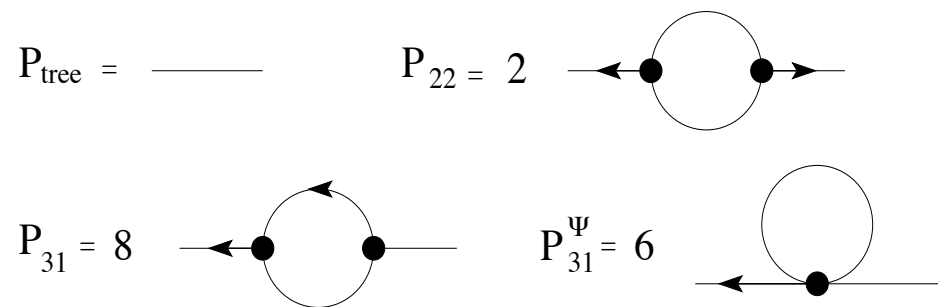
This gives in turns the density 2-pt correlation function, or the density power spectrum:

$$P(k) = P_{\text{tree}}(k) + P_{1\text{loop}}(k) + \dots$$

$$P_{\text{tree}} = P_L$$

$$P_{1\text{loop}} = P_{22} + P_{31} + P_{31}^{\Psi}$$

Diagrams:



new diagram

## Relative deviations from $\Lambda$ CDM for the power spectrum $P(k)$

### a) $f(R)$ models

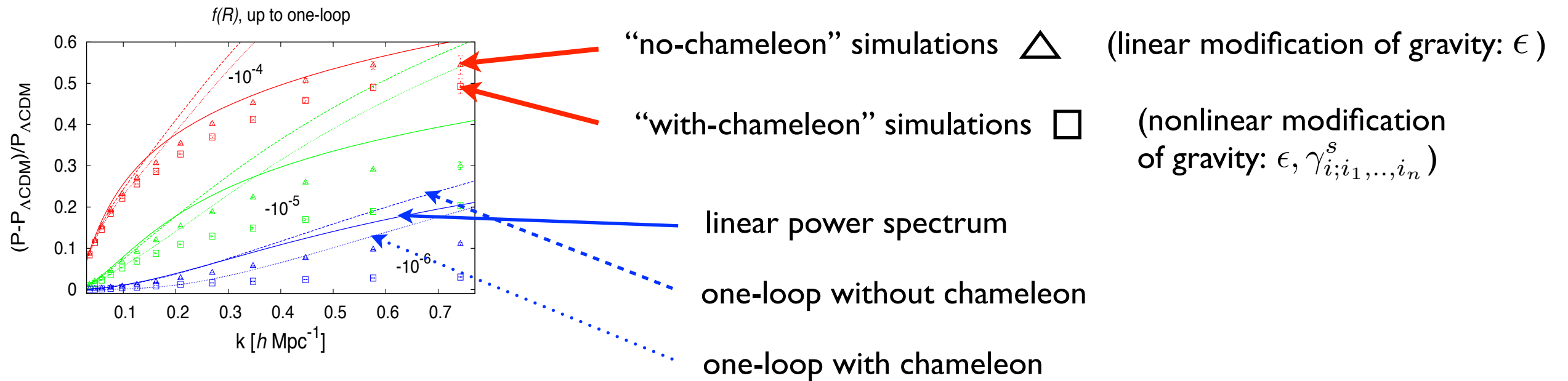
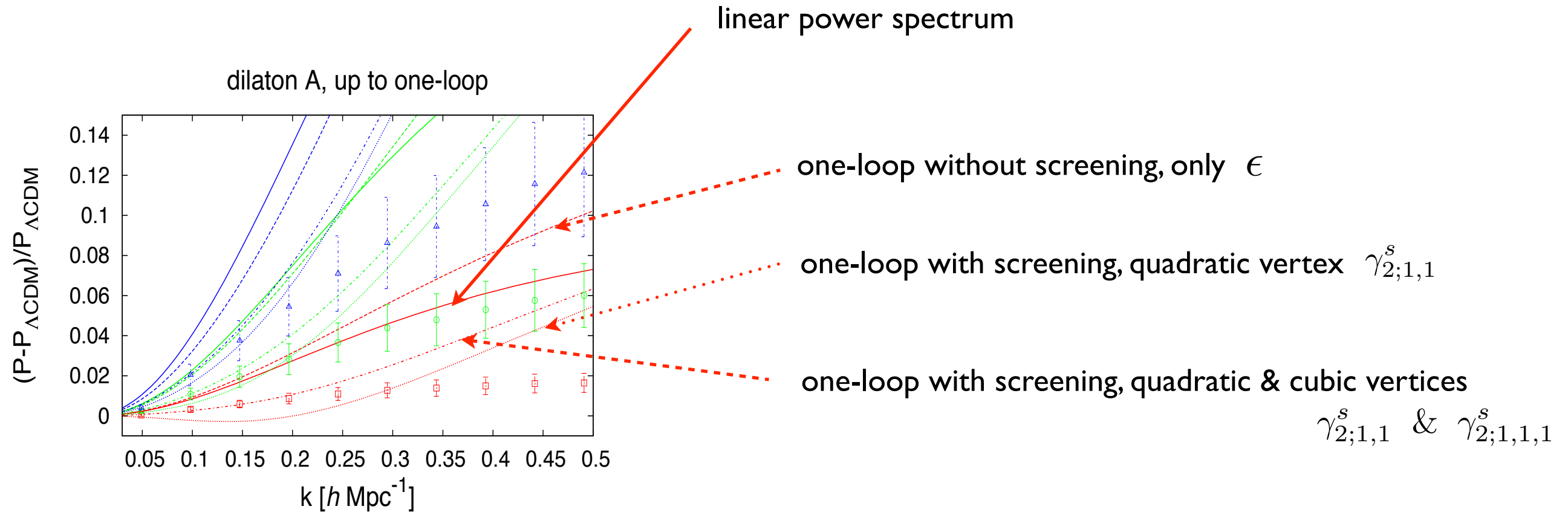


FIG. 3: Relative deviation from  $\Lambda$ -CDM of the power spectrum in  $f(R)$  theories, at redshift  $z = 0$ , for  $n = 1$  and  $f_{R0} = -10^{-4}, -10^{-5}$ , and  $-10^{-6}$ . In each case, the triangles and the squares are the results of the “no-chameleon” and “with-chameleon” simulations from [25], respectively. We plot the relative deviation of the linear power (solid line), of the one-loop power without “chameleon” effect ( $\gamma_{2;1,1}^s = \gamma_{2;1,1,1}^s = 0$ ) (dashed line), and with lowest-order “chameleon” effect ( $\gamma_{2;1,1}^s \neq 0, \gamma_{2;1,1,1}^s = 0$ ) (dotted line).

- ◆ Including the **quadratic vertex**  $\gamma_{2;1,1}^s$  gives the first sign of the **chameleon** effect.
- ◆ The cubic vertex makes no significant change.
- ◆ Going to 1-loop does not increase much the range of scales.

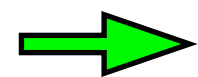
## b) Scalar field models



◆ Including the **quadratic vertex** gives the first sign of the **screening** effect.

◆ This can “**over-correct**” the deviation from ΛCDM and give a power spectrum that is smaller than the ΛCDM one. (The linear term speeds up the collapse, but the quadratic term slows down and would halt the collapse before reaching high densities.)

◆ The cubic vertex corrects for the “over-screening”.



gradual **convergence** of higher orders on perturbative scales

◆ Going to 1-loop increases somewhat the range of scales.

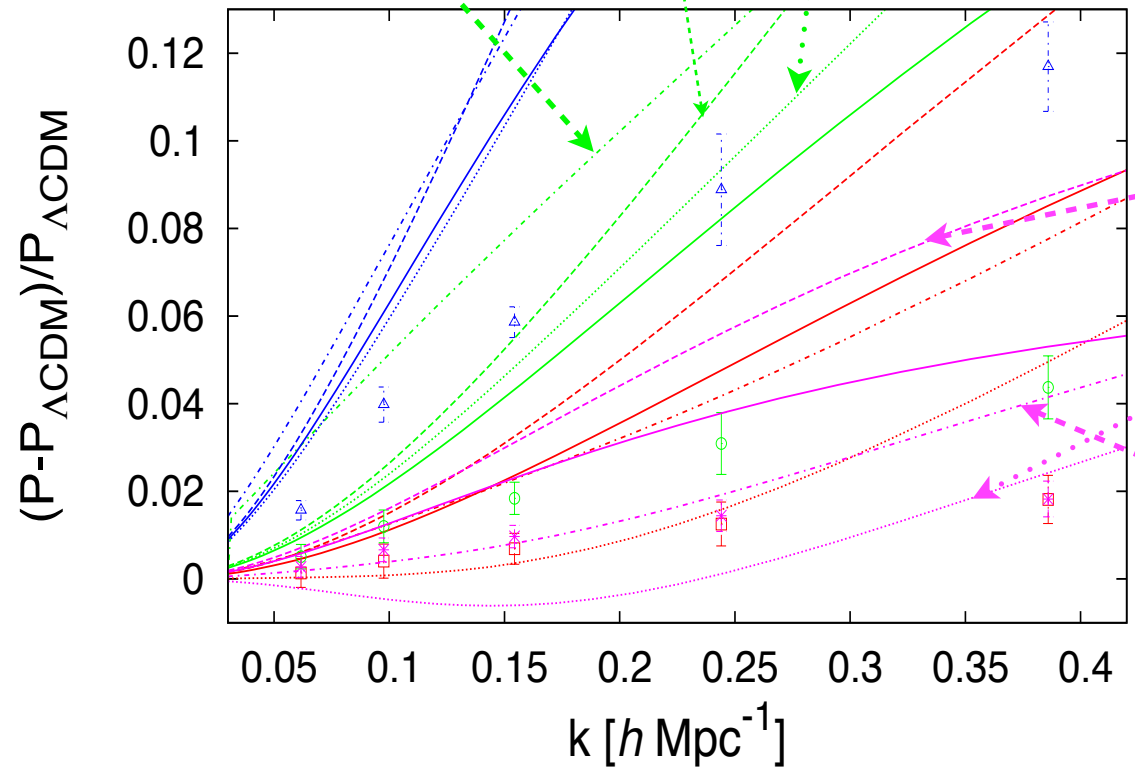
bad convergence

$\epsilon$  &  $\gamma_{2;1,1}^s$  &  $\gamma_{2;1,1,1}^s$

$\epsilon$

$\epsilon$  &  $\gamma_{2;1,1}^s$

symmetron A, up to one-loop



good convergence

$\epsilon$   
 $\epsilon$  &  $\gamma_{2;1,1}^s$   
 $\epsilon$  &  $\gamma_{2;1,1}^s$  &  $\gamma_{2;1,1,1}^s$

◆ For some models, going up to the cubic vertex can degrade the analytical predictions !

➡ bad convergence of higher orders

◆ This corresponds to models where the coupling functions are **singular**.

$$\beta(a) = \beta_0 \left[ 1 - \left( \frac{a_s}{a} \right)^3 \right]^{\hat{n}}$$

$$m(a) = m_0 \left[ 1 - \left( \frac{a_s}{a} \right)^3 \right]^{\hat{m}}$$

$$\hat{n} = 0.25, \quad \hat{m} = 0.5$$

# **Spherical collapse**




To go **beyond** 1-loop standard **perturbation theory**, we wish to combine the perturbative expansion with a halo model. We need to take into account the impact of modified gravity on the halo mass function


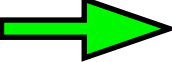
 study how the spherical collapse is modified

5th force:  $\ddot{r} = -\frac{\partial\Psi_N}{\partial r} - \frac{\partial\Psi_A}{\partial r}$  (for f(R) and dilaton models)

normalized radius  $y(t)$ :  $y(t) = \frac{r(t)}{a(t)q}$  with  $q = \left(\frac{3M}{4\pi\bar{\rho}_0}\right)^{1/3}$ ,  $y(t=0) = 1$   $\delta(< r) = y^{-3} - 1$

$$\frac{\partial^2 y}{\partial \eta^2} + \frac{1 - 3w\Omega_{de}}{2} \frac{\partial y}{\partial \eta} + \frac{\Omega_m}{2} (y^{-3} - 1) y = \frac{-3\Omega_m y}{8\pi\mathcal{G}\bar{\rho}r} \frac{\partial\Psi_A}{\partial r}$$

 in GR, all shells evolve independently before shell crossing

 the 5th force depends on the profile  
 all shells are coupled

Simplifying approximation: use an **ansatz for the density profile**, parameterized by the density contrast of the mass-shell of interest:

typical profile of rare events  
(neglecting nonlinear distortions)

$$\delta(x) = \frac{\delta_M}{\sigma_{x_M}^2} \int_{V_M} \frac{d\mathbf{x}'}{V_M} \xi_L(\mathbf{x}, \mathbf{x}')$$

## a) $f(R)$ models

Normalized fluctuation of the Ricci scalar:

$$\delta R = 8\pi\mathcal{G}\bar{\rho} \alpha(x)$$

eq. of motion for the shell M:

$$\frac{d^2 y_M}{d\eta^2} + \frac{1 - 3w\Omega_{\text{de}}}{2} \frac{dy_M}{d\eta} + \frac{\Omega_m}{2} (y_M^{-3} - 1) y_M = \frac{-\Omega_m y_M}{2} \int_0^{x_M} \frac{dx x^2}{x_M^3} (\delta - \alpha)$$

constraint eq. for the new field:

$$\frac{d^2 \alpha}{dx^2} + \frac{2}{x} \frac{d\alpha}{dx} - \frac{(n+2)\Omega_{m0}}{\Omega_{m0}(1+\alpha) + 4\Omega_{\Lambda 0} a^{-3}} \left( \frac{d\alpha}{dx} \right)^2 = a^2 m_0^2 \left( \frac{\Omega_{m0} a^{-3} (1+\alpha) + 4\Omega_{\Lambda 0}}{\Omega_{m0} + 4\Omega_{\Lambda 0}} \right)^{n+2} (\alpha - \delta)$$

◆ Large scales: weak-field (linear) regime,  $\frac{d}{dx} \rightarrow 0$ ,  $\alpha \rightarrow \delta$   GR

◆ High density: strong-field (nonlinear) regime,  $\delta \rightarrow \infty$ ,  $\alpha \rightarrow \delta$   GR

 chameleon mechanism due to the nonlinearity.

Because of the 5th force, the **linear density threshold** to reach a given nonlinear density contrast (200) is **smaller** than for LCDM.

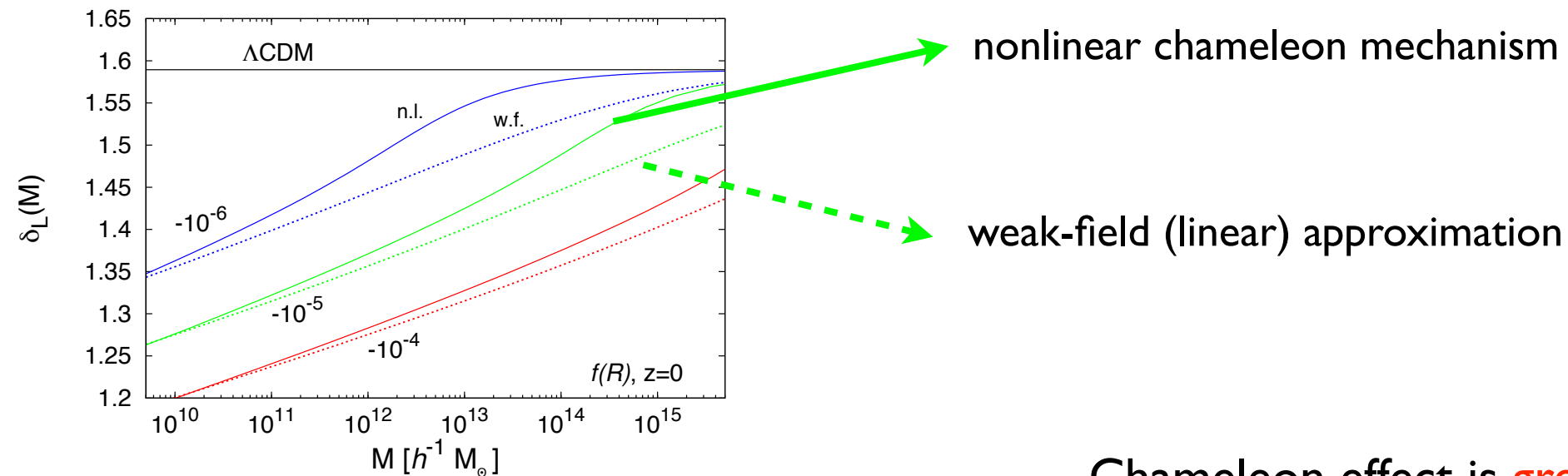


FIG. 6: Linear density threshold  $\delta_L(M)$ , associated with a nonlinear density contrast  $\delta = 200$ , for  $f(R)$  theories at  $z = 0$ . The dotted lines (w.f.) correspond to the weak-field limit (108) and the solid lines (n.l.) to the fully nonlinear constraint (106).

Chameleon effect is **greater for large masses**, where nonlinearities can overcome spatial gradients.


- ◆ The linear density threshold becomes **mass-dependent**:  $\delta_L(M)$
- ◆ The deviation from LCDM diminishes at high mass.
- ◆ The **nonlinear chameleon** effect decreases the deviation from LCDM. It is more efficient for large masses.

## b) Scalar field models

“Normalized” scalar field:  $\alpha(x) = a[\varphi(x)]$

eq. of motion for the shell M: 
$$\frac{d^2 y_M}{d\eta^2} + \frac{1 - 3w\Omega_{de}}{2} \frac{dy_M}{d\eta} + \frac{\Omega_m}{2} (y_M^{-3} - 1) y_M = \frac{-9\Omega_m a \beta_\alpha^2 y_M}{m_\alpha^2 \alpha^4 x_M} \frac{\partial \alpha}{\partial x}$$

Klein-Gordon eq.: 
$$\frac{d^2 \alpha}{dx^2} + \frac{2}{x} \frac{d\alpha}{dx} + \left[ \frac{d \ln \beta_\alpha}{d\alpha} - 2 \frac{d \ln m_\alpha}{d\alpha} - \frac{4}{\alpha} \right] \left( \frac{d\alpha}{dx} \right)^2 = \frac{m_\alpha^2 \alpha^4}{3a} \left[ 1 + \delta - \frac{a^3}{\alpha^3} \right]$$

◆ Large scales: weak-field (linear) regime,  $\frac{d}{dx} \rightarrow 0$ ,  $\alpha \rightarrow a(1 + \delta)^{-1/3}$   GR

◆ High density: strong-field (nonlinear) regime,  $\delta \rightarrow \infty$ ,  $\alpha \rightarrow a\delta^{-1/3}$

dilaton models:  $\frac{\beta_\alpha^2}{m_\alpha^2} \rightarrow 0$

symmetron models:  $\alpha \rightarrow a_s$

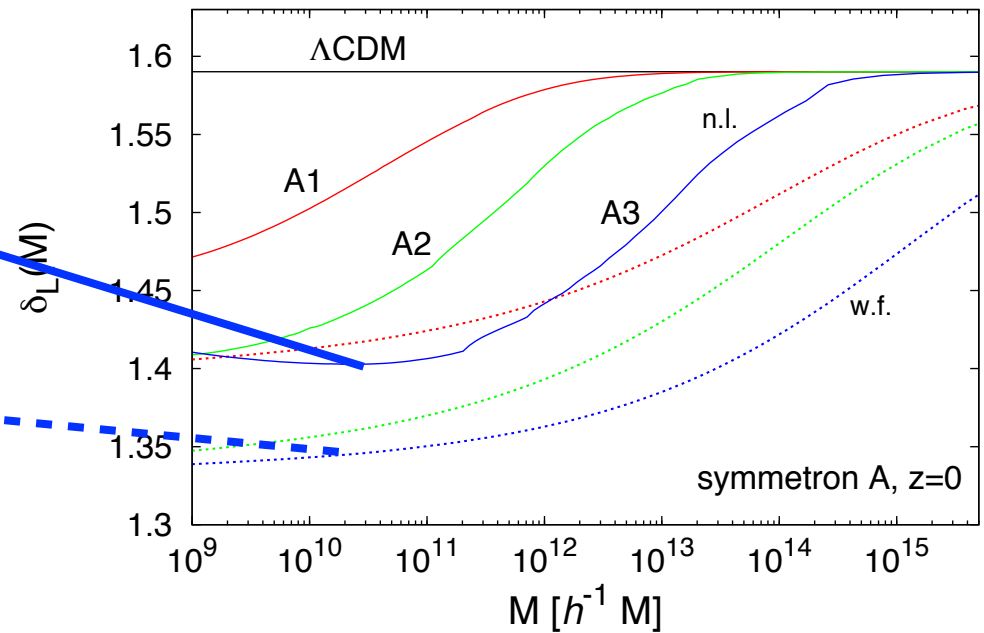
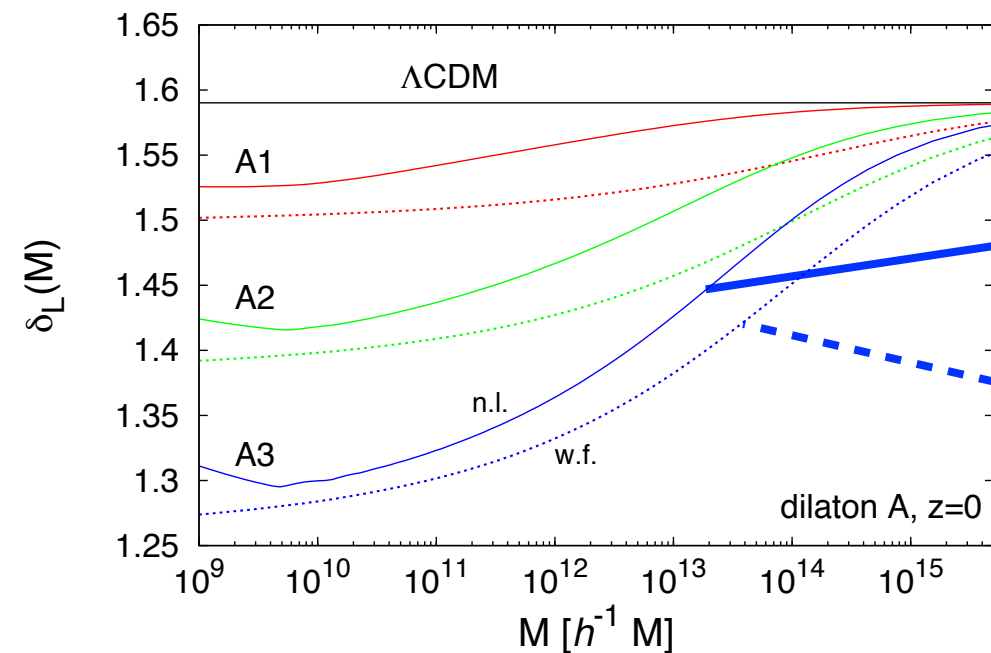
 GR

 screening mechanism due to the nonlinearity.

Because of the 5th force, the **linear density threshold** to reach a given nonlinear density contrast (200) is **smaller** than for LCDM.

dilaton models

symmetron models



nonlinear screening

weak-field approx.

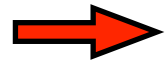
- ◆ Again, nonlinearities (screening) decrease the deviation from GR.
- ◆ The rate of convergence to GR at high mass depends on the model (very efficient for symmetron, very nonlinear models).
- ◆ Contrary to  $f(R)$  models, at low mass we do not converge to weak-field prediction but to GR.

## c) K-mouflage models

Trajectories in physical coordinates:

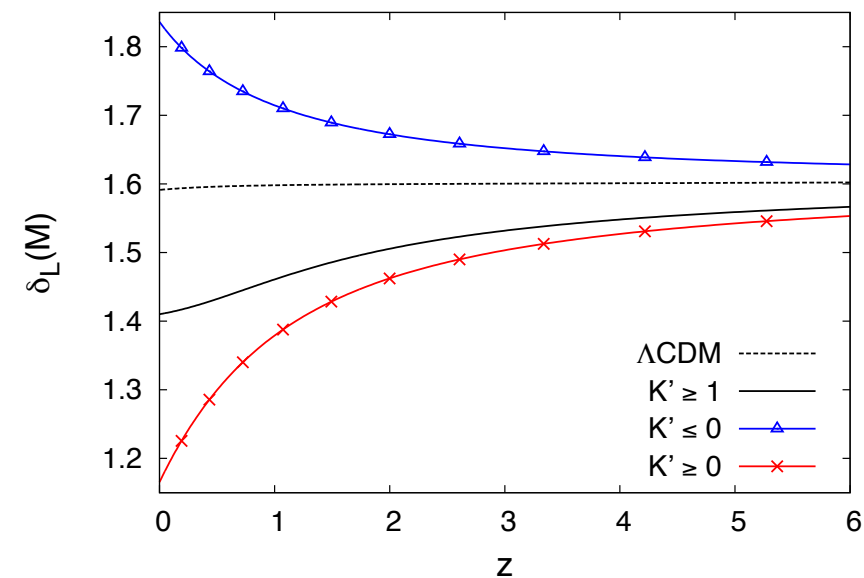
$$\ddot{\mathbf{r}} + \frac{d \ln \bar{A}}{dt} \dot{\mathbf{r}} - \left( \frac{\ddot{a}}{a} + \frac{\dot{a}}{a} \frac{d \ln \bar{A}}{dt} \right) \mathbf{r} = -\nabla_r (\Psi_N + \ln A)$$

Scale-independence



the motions of **different mass shells are decoupled**,  
as in LCDM (before shell-crossing)

linear density contrast threshold:



## d) Halo mass function

$$M \rightarrow \infty : \ln[n(M)] \sim -\frac{\delta_c(M)^2}{2\sigma(M)^2}$$

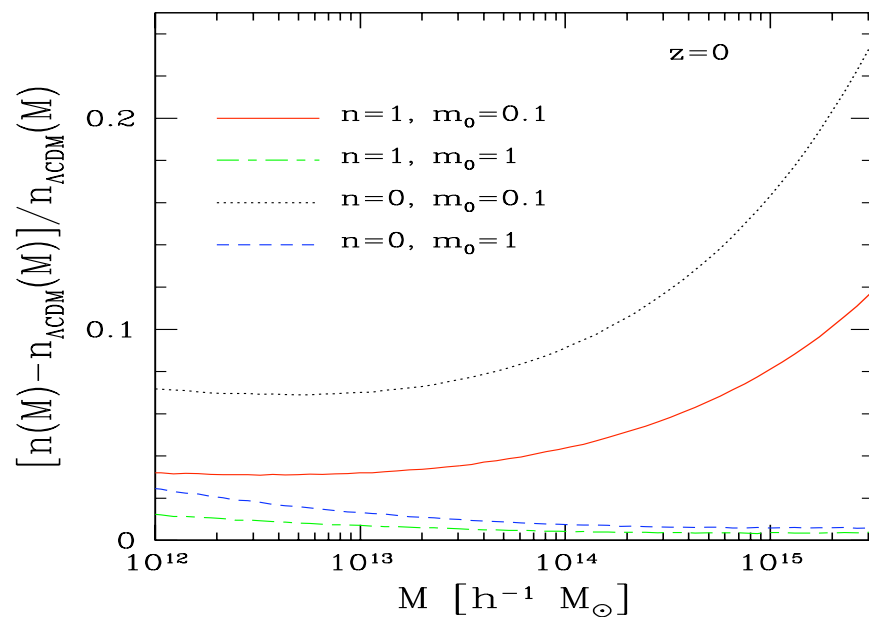
with

$$\delta_c(M) = \mathcal{F}_q^{-1}(200)$$

Use the Press-Schechter scaling:

$$n(M) \frac{dM}{M} = \frac{\bar{\rho}_m}{M} f(\nu) \frac{d\nu}{\nu} \quad \text{with}$$

$$\nu = \frac{\delta_c(M)}{\sigma(M)}$$



Relative deviation of the mass function

case where  $\epsilon(k, a) > 0$

## e) Probability distribution of the density contrast

From the spherical dynamics we can also obtain the PDF of the density contrast within spherical cells, in the weakly non-linear regime.

Introduce the cumulant generating function (Laplace transform):

$$e^{-\varphi(y)/\sigma_x^2} \equiv \langle e^{-y\delta_x/\sigma_x^2} \rangle = \int_{-1}^{\infty} d\delta_x e^{-y\delta_x/\sigma_x^2} \mathcal{P}(\delta_x)$$

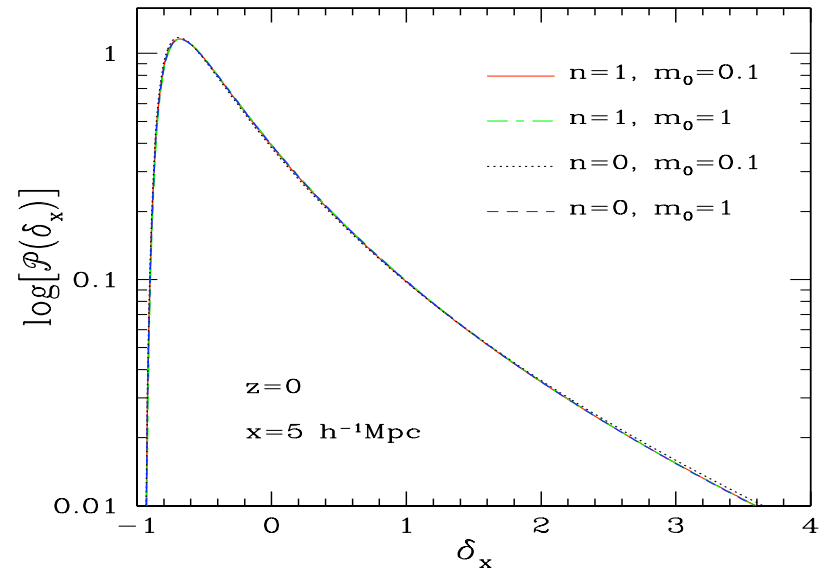
$$e^{-\varphi(y)/\sigma_x^2} = (\det C_{\delta_L \delta_L}^{-1})^{1/2} \int \mathcal{D}\delta_L e^{-S[\delta_L]/\sigma_x^2} \quad \text{where} \quad S[\delta_L] = y \delta_x[\delta_L] + \frac{\sigma_x^2}{2} \delta_L \cdot C_{\delta_L \delta_L}^{-1} \cdot \delta_L$$

On large scales, we obtain:

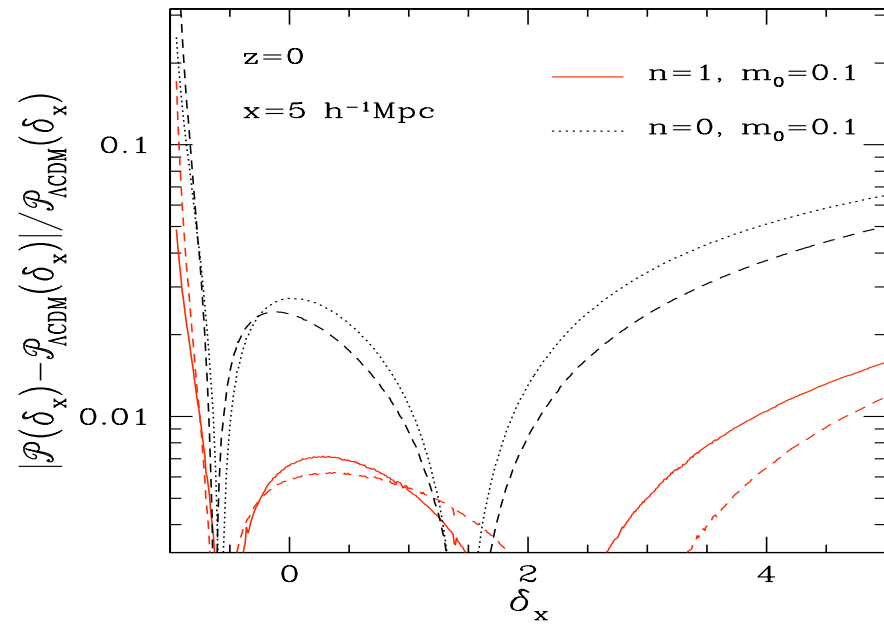
$$\sigma_x \rightarrow 0 : \quad \varphi(y) \rightarrow \min_{\delta_L} S[\delta_L]$$

For spherical cells, we can look for the spherical minimum (saddle-point)





Probability distribution function



Relative deviation

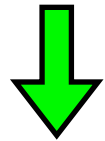
case where  $\epsilon(k, a) > 0$

# **Matter power spectrum**

As in the halo model (but from a Lagrangian point of view), decompose the power spectrum as

$$P(k) = P_{2H}(k) + P_{1H}(k)$$

“2-halo term”

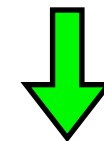


perturbative contribution

$$P_{2H}(k) \simeq F_{2H}(1/k) P_{\text{pert}}(k)$$

(high-k behavior improved by going beyond standard perturbation theory)

“1-halo term”



nonperturbative contribution

$$P_{1H}(k) = \int_0^\infty \frac{d\nu}{\nu} f(\nu) \frac{M}{\bar{\rho}(2\pi)^3} \left( \tilde{u}_M(k)^2 - \tilde{W}(k q_M)^2 \right)$$

halo mass function

halo density profile

(low-k behavior solved by counterterm)

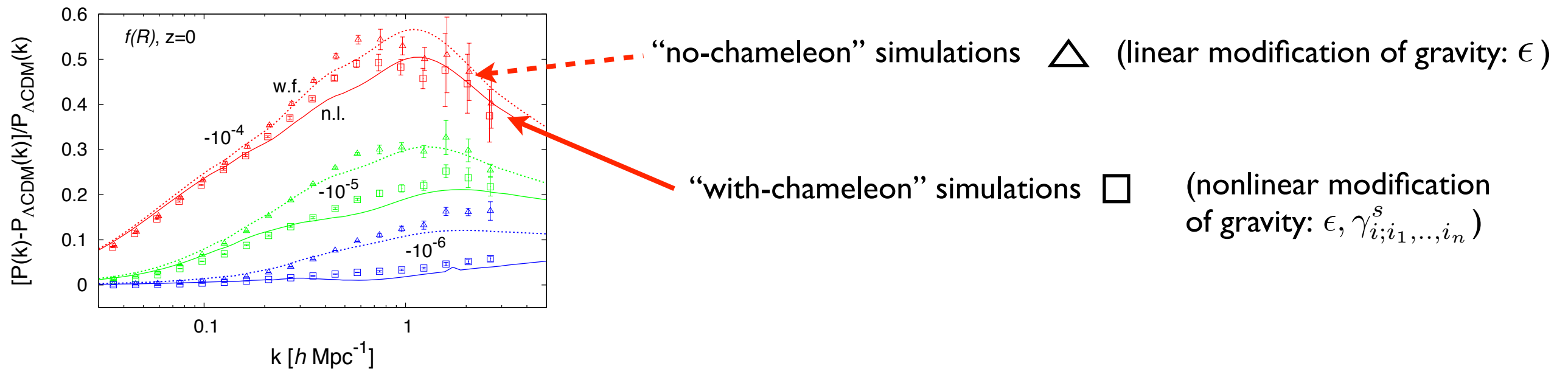
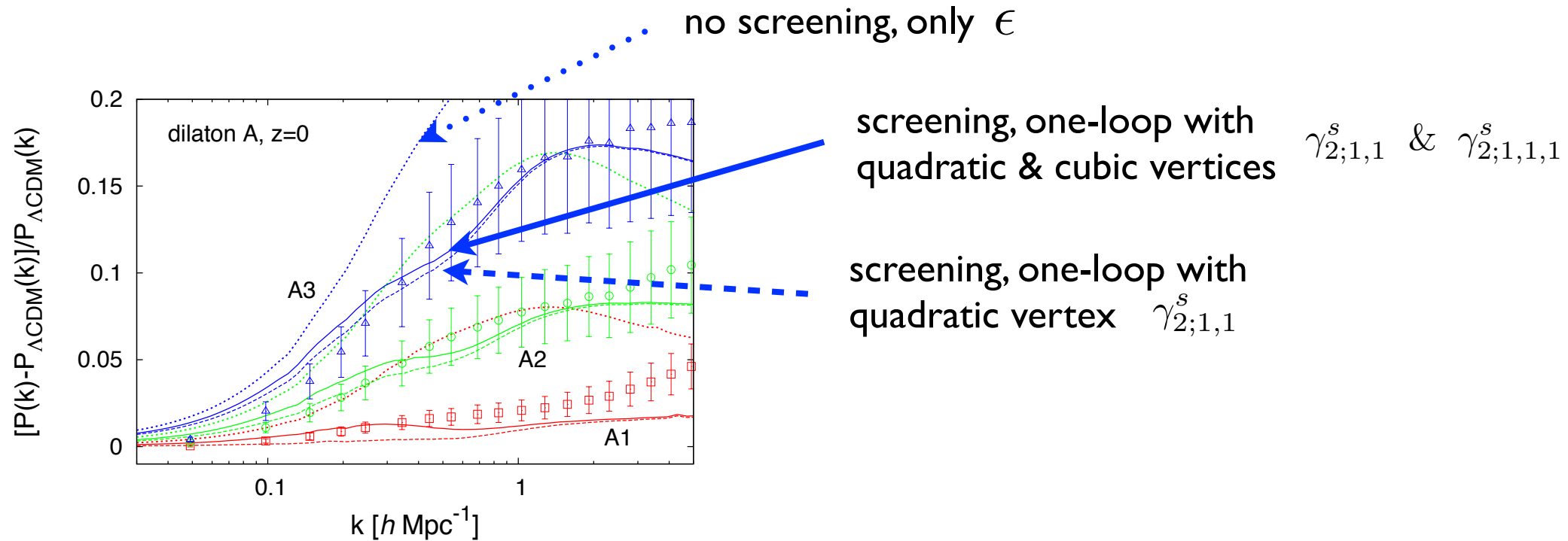


FIG. 13: Relative deviation from  $\Lambda$ -CDM of the power spectrum in  $f(R)$  theories, at redshift  $z = 0$ , for  $n = 1$  and  $f_{R_0} = -10^{-4}, -10^{-5}$ , and  $-10^{-6}$ . In each case, the triangles and the squares are the results of the “no-chameleon” and “with-chameleon” simulations from [25], respectively. We plot the relative deviation of the nonlinear power spectrum without chameleon effect (w.f., dotted lines) and with chameleon effect (n.l., solid lines).

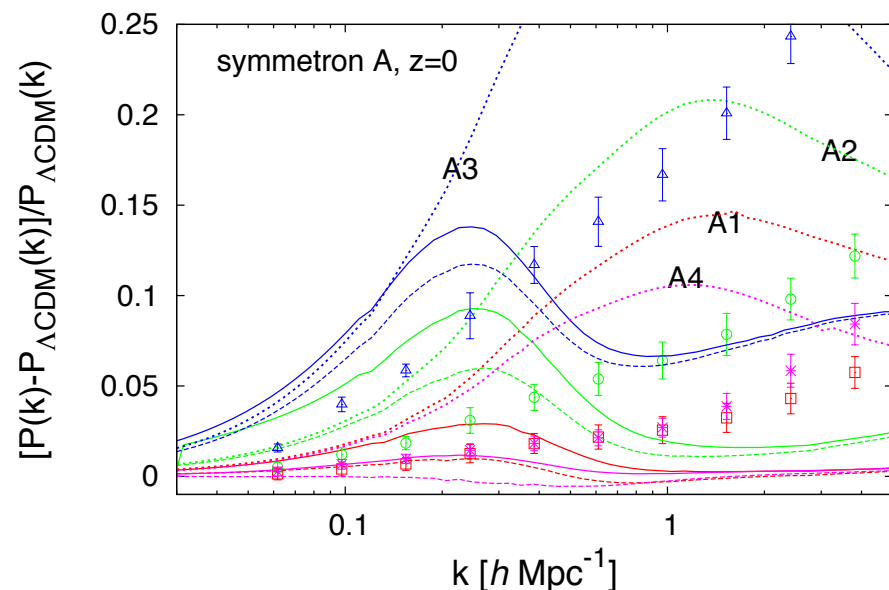
- ◆ **Reasonably good agreement** between simulations and analytical predictions, from linear to mildly nonlinear scales.
- ◆ As compared with parameterizations (PPF), the convergence to GR on small scales is **not put by hand**. It is due to the chameleon mechanism.

## b) Scalar field models

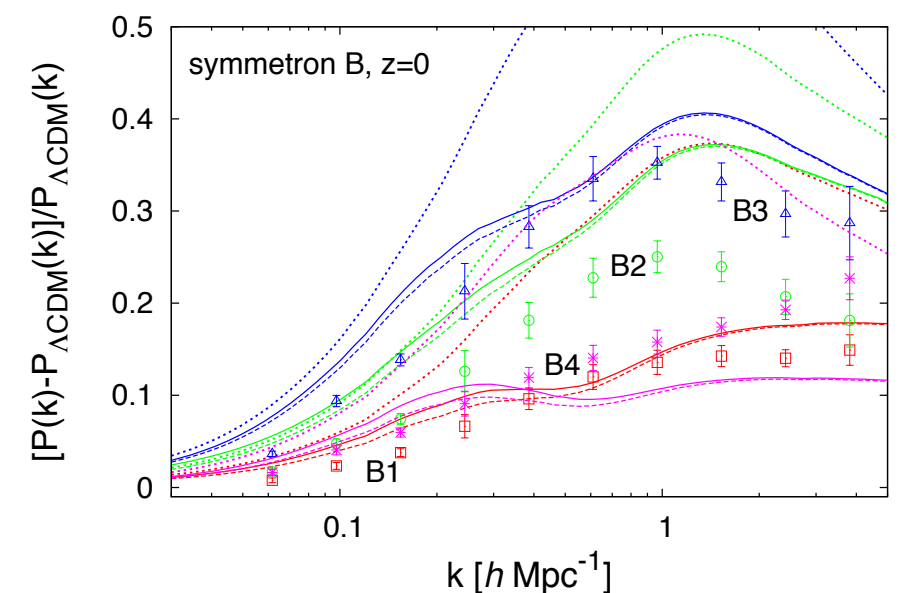
Relative deviation from  $\Lambda$ CDM for  $P(k)$



- ◆ The impact of the nonlinear screening mechanism is greater than for the  $f(R)$  models.
- ◆ Reasonably good agreement with simulations.
- ◆ Underestimate at high  $k$ , could be due to the neglect of halo profile modifications.



Bad convergence, but we can guess beforehand the problematic cases.



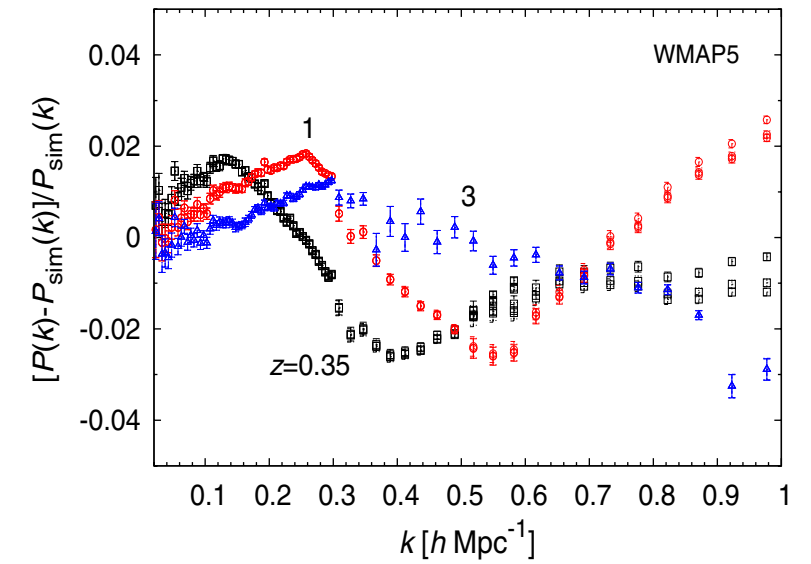
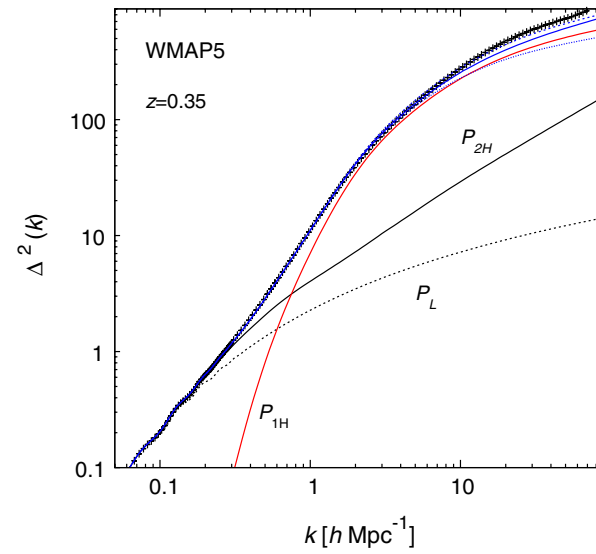
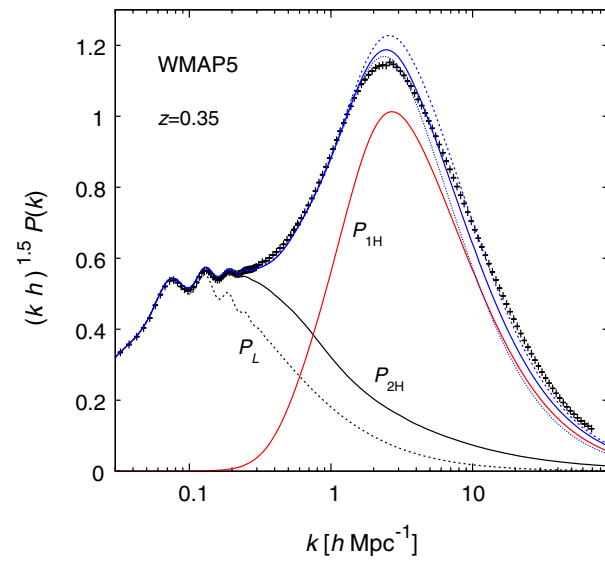
Good convergence, reasonable agreement.

# Conclusion

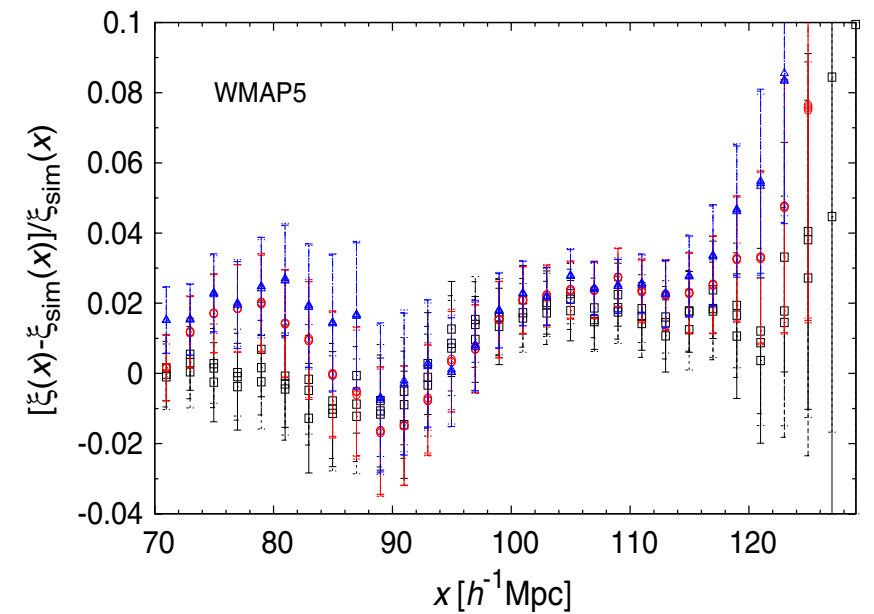
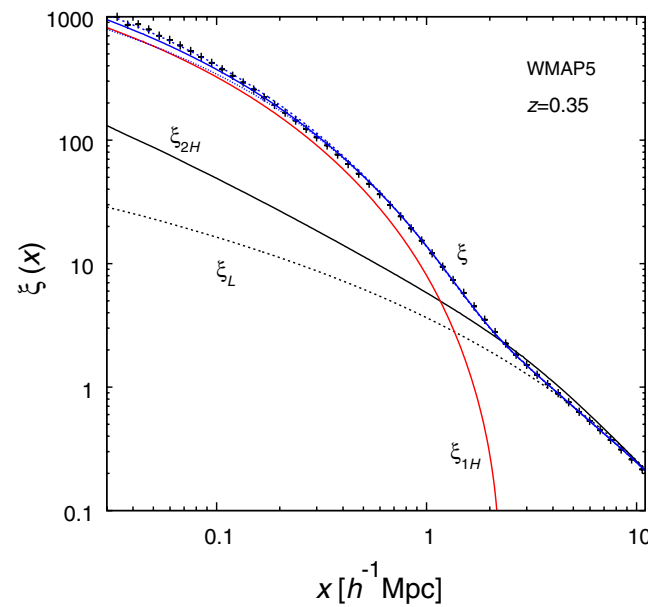
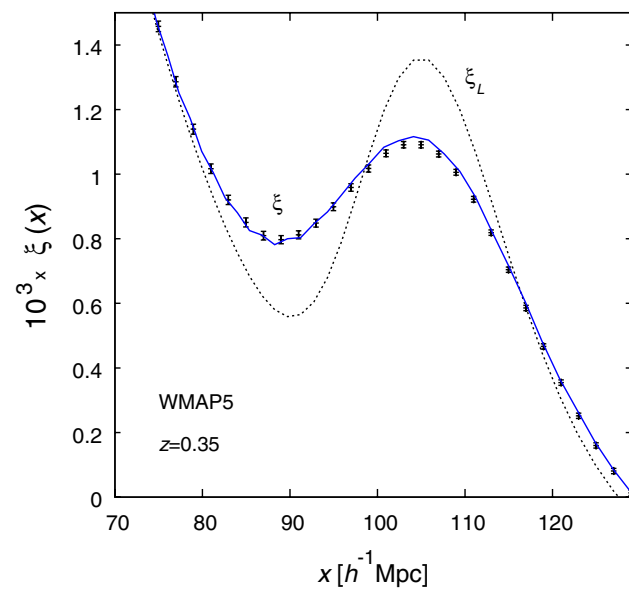
- ◆ “Modified-gravity” models introduce a **new degree of freedom** (new field).
- ◆ Using the **quasi-static approximation**, we can go back to the standard framework, defined by the matter density and velocity fields, with a modified “gravitational” potential.
  - ◆ “Standard” perturbation theory can be generalized in a direct manner. The main differences are:
    - new complex **time** and **scale dependences**.
    - new **nonlinear vertices** (the eqs. of motion are no longer quadratic), which are the first signs of nonlinear screening mechanisms.
- ◆ The spherical collapse is more complex, because of the **coupling between different shells**. Nevertheless, this can be simplified using approximate density profiles.
  - ◆ Explicit account of **nonlinear chameleon or screening** mechanisms that ensure convergence to GR in high-density environments.
- ◆ By combining perturbation theory and halo model (spherical collapse), one can obtain **reasonably good predictions** up to mildly nonlinear scales, for models that are not too singular.
- ◆ **Singular** models lead to **bad convergence** of perturbative expansions and low accuracy of analytical predictions. Fortunately, these cases can be detected before hand.
- ◆ To handle difficult cases, or to go beyond the quasi-static approximation, one may need to explicitly keep track of the new scalar field in the perturbative approach ?

# Supplements

Matter density power spectrum:



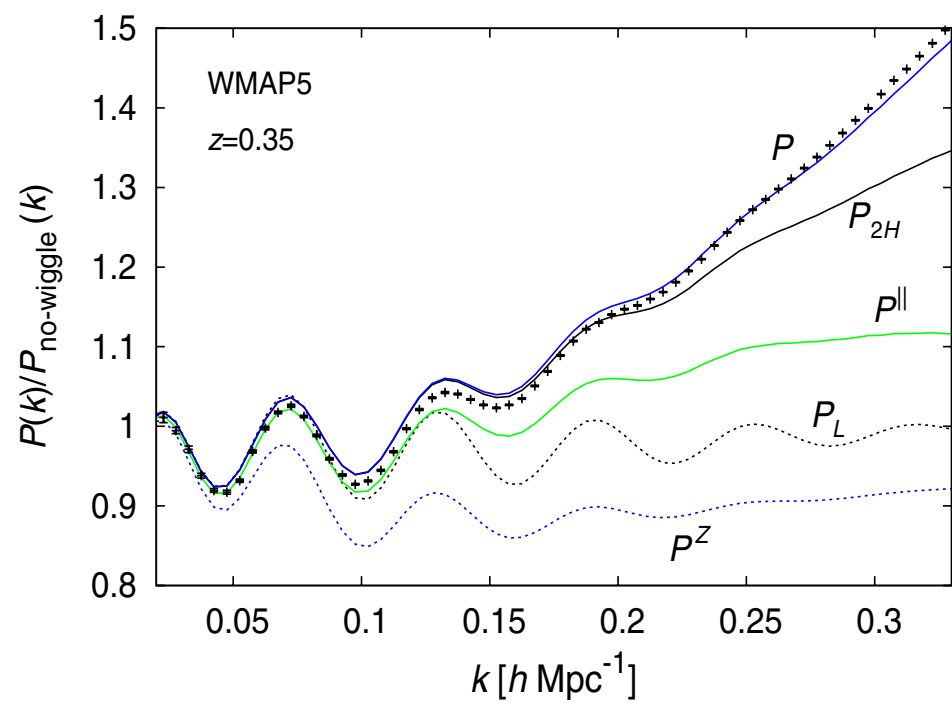
Matter density correlation function:





# BAO scales

## power spectrum



## correlation function

