The TRG and coarse-grained PT's

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Outline

- Mode coupling or, where SPT fails
- The TRG: IR, intermediate, and UV effects
- Scalar field (axion/fuzzy) DM

Mode coupling-Response functions

The nonlinear PS is a functional of the initial one (in a given cosmology and assuming no PNG):

SPT is an expansion around $P^0(q) = 0$

$$P_{ab}[P^0](\mathbf{k};\eta) = \sum_{n=1}^{\infty} \frac{1}{n!} \int d^3q_1 \cdots d^3q_n \left. \frac{\delta^n P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q}_1)\cdots\delta P^0(\mathbf{q}_n)}\right|_{P^0=0} P^0(\mathbf{q}_1)\cdots P^0(\mathbf{q}_n)$$

n=1 linear order (= “0-loop”)
n=2 “1-loop”
…

$a, \cdots, d = 1$ density
$a, \cdots, d = 2$ velocity div.

Mode coupling-Response functions

Let's instead expand around a reference PS: $P^0(q) = \bar{P}^0(q)$

$$
\begin{aligned}
P_{ab}[P^0](\mathbf{k};\eta) &= P_{ab}[\bar{P}^0](\mathbf{k};\eta) \\
&\quad + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^3q_1 \cdots d^3q_n \; \left. \frac{\delta^n P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q}_1)\cdots\delta P^0(\mathbf{q}_n)} \right|_{P^0=\bar{P}^0} \delta P^0(\mathbf{q}_1)\cdots\delta P^0(\mathbf{q}_n)\,, \\
&= P_{ab}[\bar{P}^0](\mathbf{k};\eta) + \int \frac{dq}{q}\, K_{ab}(k,q;\eta)\,\delta P^0(q) + \cdots\,,
\end{aligned}
\qquad \delta P^0(\mathbf{q}) \equiv P^0(\mathbf{q}) - \bar{P}^0(\mathbf{q})
$$

Linear response function: $K_{ab}(k,q;\eta) \equiv q^3 \int d\Omega_{\mathbf{q}} \left. \frac{\delta P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q})} \right|_{P^0=\bar{P}^0}$

Non-perturbative (gets contributions from all SPT orders)

Key object for more efficient interpolators ?

IR and UV screening

Sensitivity of the nonlinear PS at scale k on a change of the initial PS at scale q:

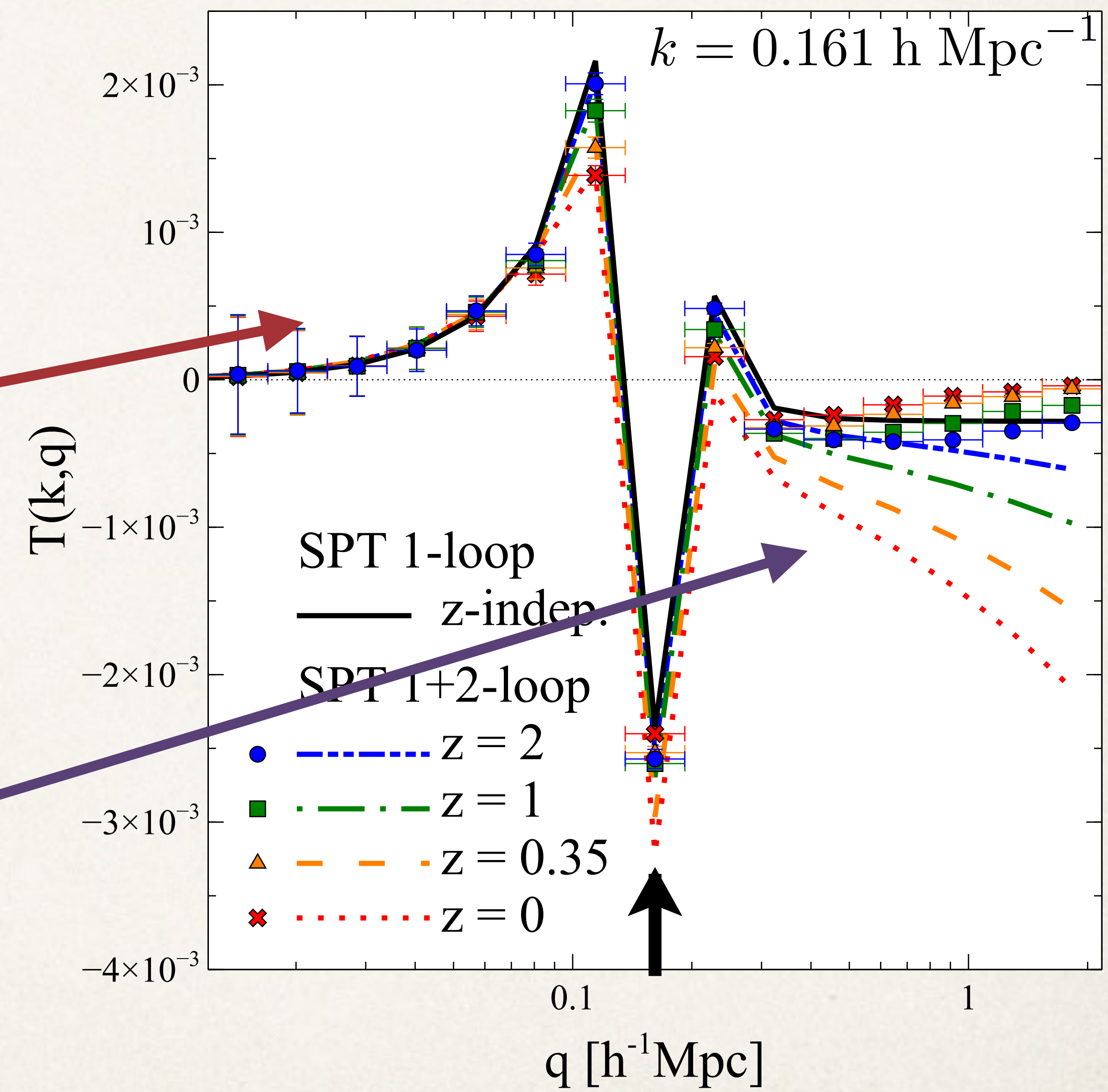
$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$

IR: "Galilean invariance"

$K(k,q;z) \sim q^3$

Peloso, MP 1302.0223

PT overpredicts the effect of UV scales on intermediate ones

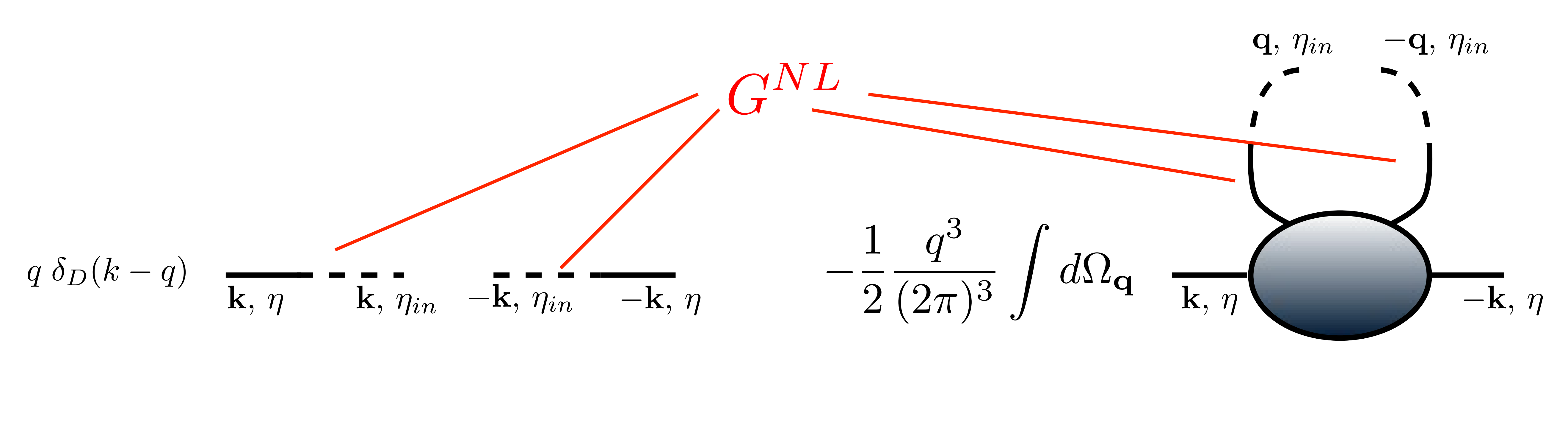


Nishimichi, Bernardeau, Taruya 1411.2970

… Little, Weinberg, Park, 1991

The non-perturbative LRF

$$K_{ab}(k,q;\eta) = q\,\delta_D(k-q)\,G_{ac}(k;\eta,\eta_{in})u_c\,G_{bd}(k;\eta,\eta_{in})u_d$$
$$-\frac{1}{2}\frac{q^3}{(2\pi)^3}\int d\Omega_{\mathbf{q}}\,\langle\varphi_a(\mathbf{k};\eta)\chi_c(-\mathbf{q};\eta_{in})\chi_d(\mathbf{q};\eta_{in})\varphi_b(-\mathbf{k};\eta)\rangle'_c\,u_c u_d\,,$$



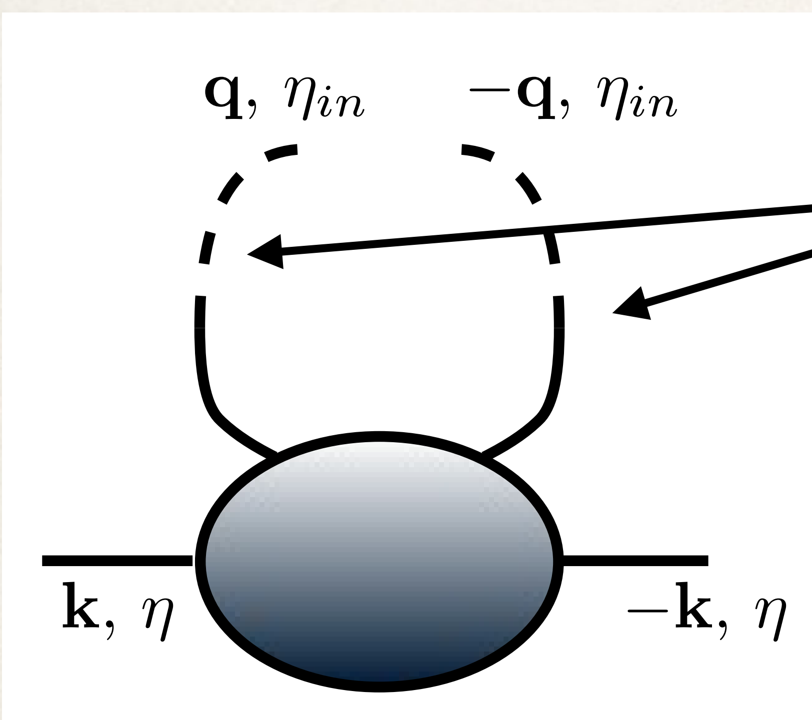
$$G_{ab}(k;\eta,\eta_{in}) = \langle\frac{\delta\varphi_a(\mathbf{k},\eta)}{\delta\varphi_b(\mathbf{k},\eta_{in})}\rangle' = -i\langle\varphi_a(\mathbf{k},\eta)\chi_b(-\mathbf{k},\eta_{in})\rangle'$$

methods from Matarrese, MP, '07

UV screening

The effect of virialized structures on larger scales is screened (Peebles '80, Baumann et al 1004.2488, Blas et al 1408.2995).

However, the departure from the PT predictions starts at small q's: is it only a virialization effect?



$e^{-\frac{q^2\sigma_v^2}{2}}$ damped propagators!

(compare SPT: linear propagator g=O(1))

memory of initial substructures is largely lost

UV lessons

- SPT fails when loop momenta become higher than the nonlinear scale ($q \gtrsim 0.4$ h/Mpc)
- The real response to modifications in the UV regime is mild
- Most of the cosmology dependence is on intermediate scales

The nonlinear PS

$$\boxed{P_{ab}^{NL}(k,z) = G_{ac}(k,z)G_{bd}(k,z)P_{cd}^{lin}(k,z) + P_{ab}^{MC}(k,z)}$$

Mostly IR physics

IR, intermediate and UV physics

The TRG

apply the equation of motion

$$\left(\delta_{ab}\,\partial_\eta + \Omega_{ab}\right)\varphi_b\left(\mathbf{k},\,\eta\right) = I_{\mathbf{k},\,\mathbf{q}_1,\,\mathbf{q}_2}\,\mathrm{e}^{\eta}\gamma_{abc}\left(\mathbf{q}_1,\,\mathbf{q}_2\right)\varphi_b\left(\mathbf{q}_1,\,\eta\right)\varphi_c\left(\mathbf{q}_2,\,\eta\right) - h_a\left(\mathbf{k},\,\eta\right)$$

to the (nonlinear) PS

$$\partial_\eta P_{ab}\left(k;\,\eta,\,\eta\right) = \Big[-\,\Omega_{ac}\,P_{cb}\left(k;\,\eta,\,\eta\right) + \mathrm{e}^{\eta}\,I_{\mathbf{k},\,\mathbf{q}_1,\,\mathbf{q}_2}\,\gamma_{acd}\left(\mathbf{q}_1,\,\mathbf{q}_2\right)\;B_{cdb}(q_1, q_2, k; \eta, \eta, \eta)$$
$$-\,\langle h_a\left(\mathbf{k},\,\eta\right)\;\varphi_b\left(-\mathbf{k},\,\eta\right)\rangle'\Big] + \left(a\;\leftrightarrow\;b\right)\;,$$

Virtues

$$\mathbf{\Omega}(\mathbf{k},\eta) = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_m(\eta)(1+\mathcal{B}(\mathbf{k},\eta)) & 2+\frac{\mathcal{H}'}{\mathcal{H}}+\mathcal{A}(\mathbf{k},\eta) \end{pmatrix} \qquad \eta = \log\left(\frac{a}{a_0}\right)$$

Time and scale-dependent growth

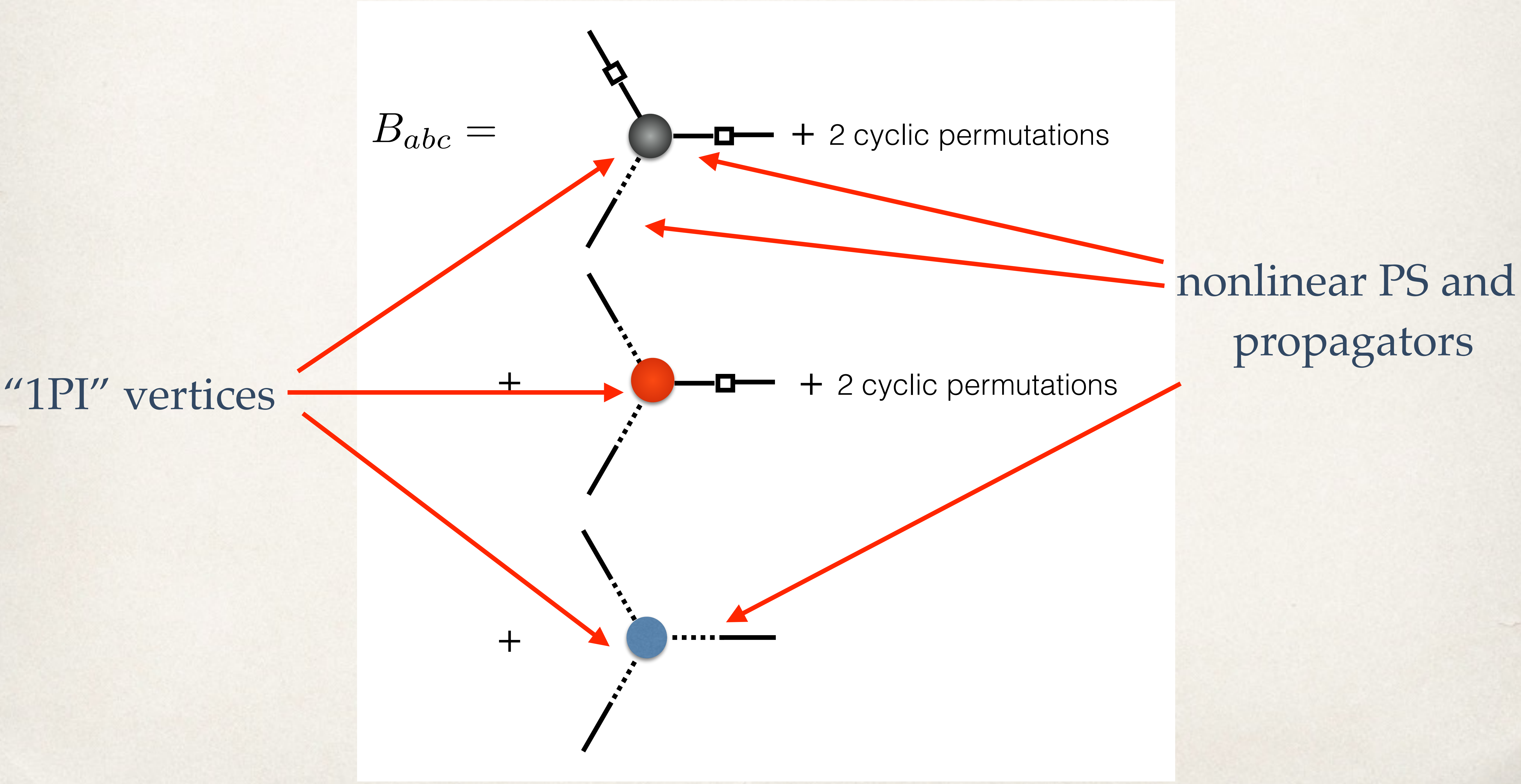
Treats the decaying mode(s) correctly

Good for multi-species (CDM+neutrinos, B+CDM, DM+Halo, modified GR…), and for PNG

“Galilean” invariant (equal time correlators)

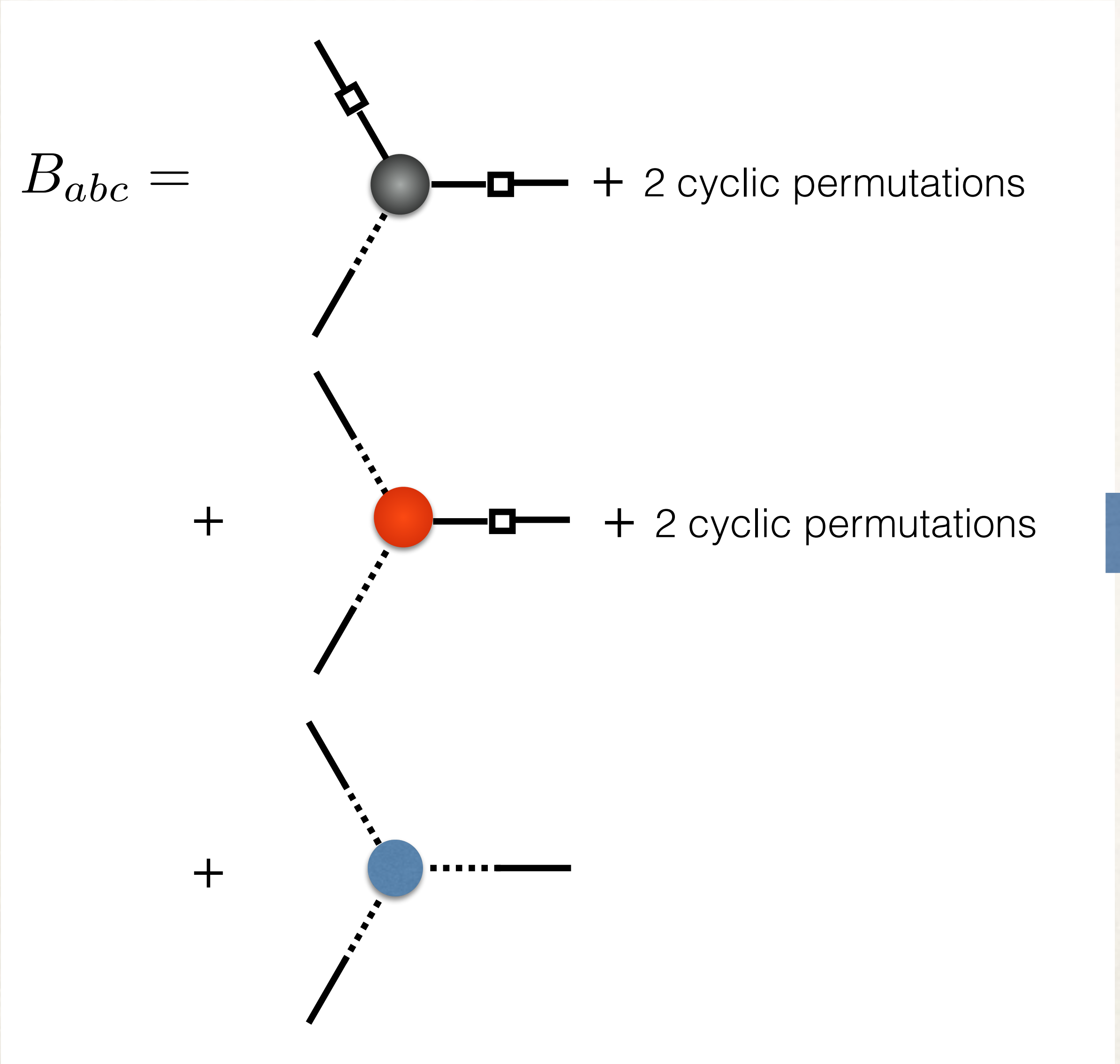
Can be fast (see later)

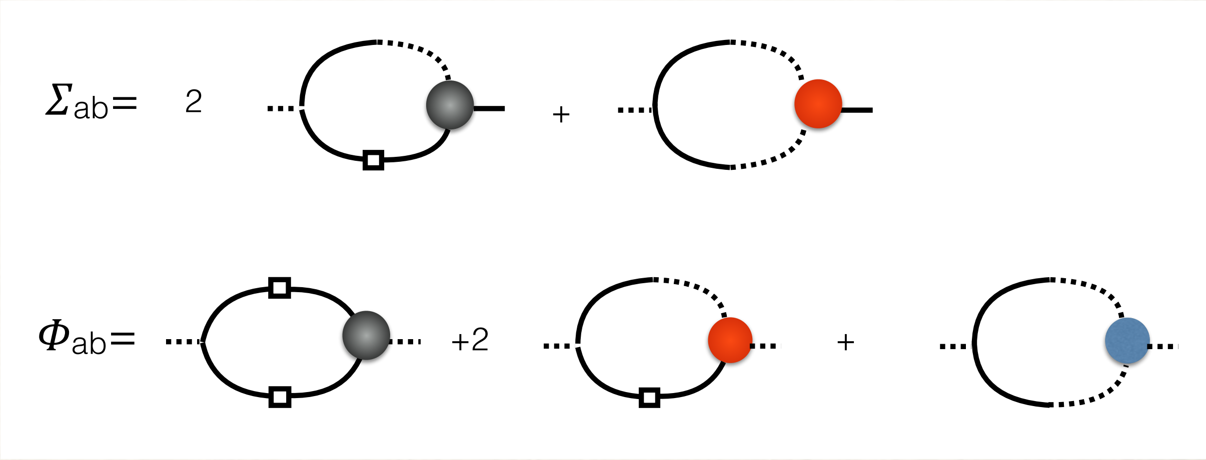
The Full Bispectrum



Closing the loops

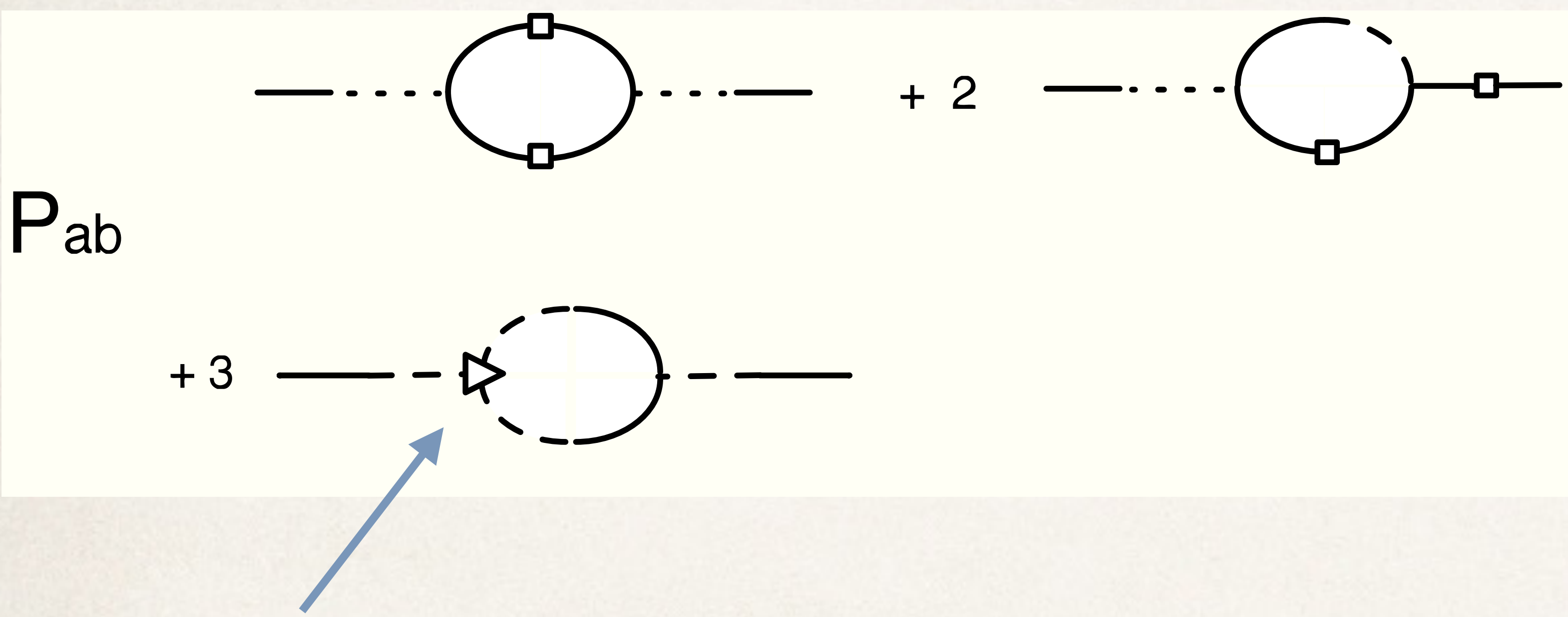
$$I_{\mathbf{k},\,\mathbf{q}_1,\,\mathbf{q}_2}\,\gamma_{acd}\,(\mathbf{q}_1,\;\mathbf{q}_2)\;\;B_{cdb}(q_1,q_2,k;\eta,\eta,\eta)$$





Simplest Truncation

Computing the bispectrum by
a truncated TRG equation (trispectrum=0) gives



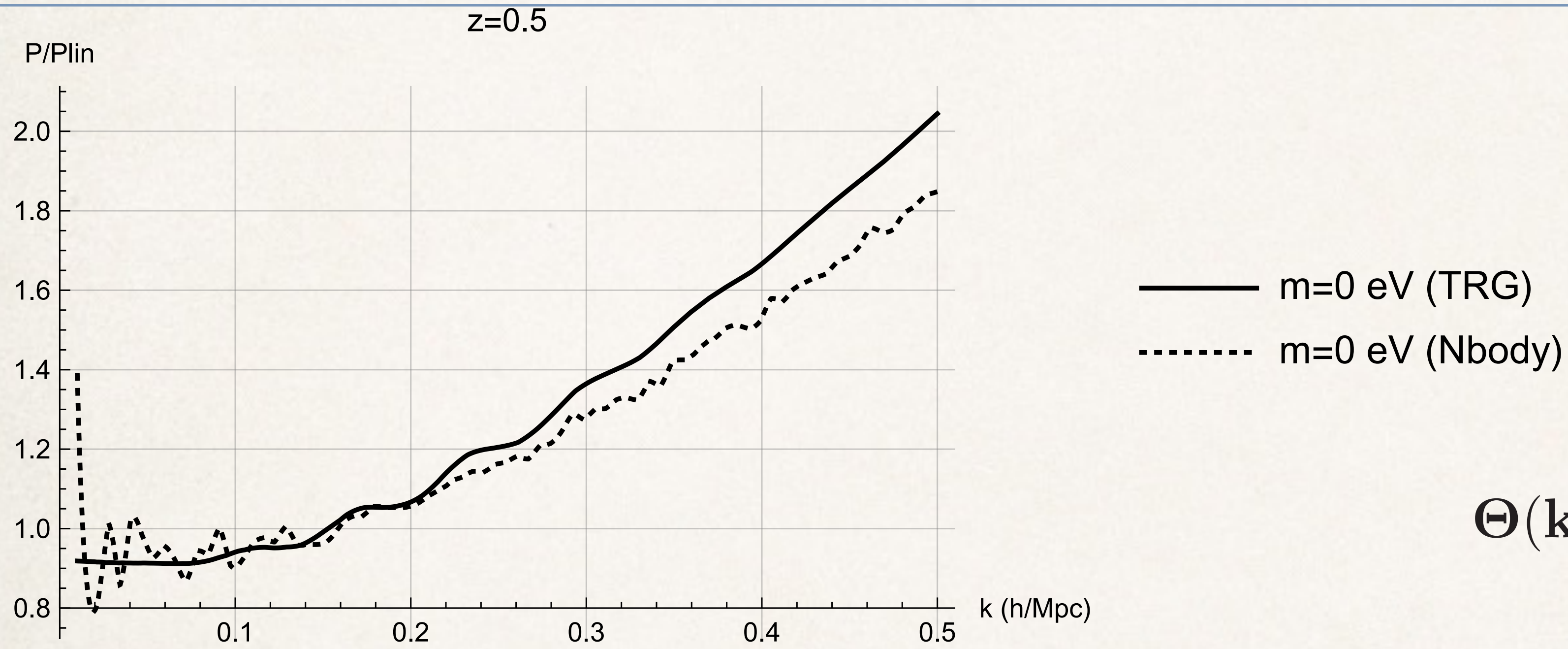
PNG

linear vertices
and propagators

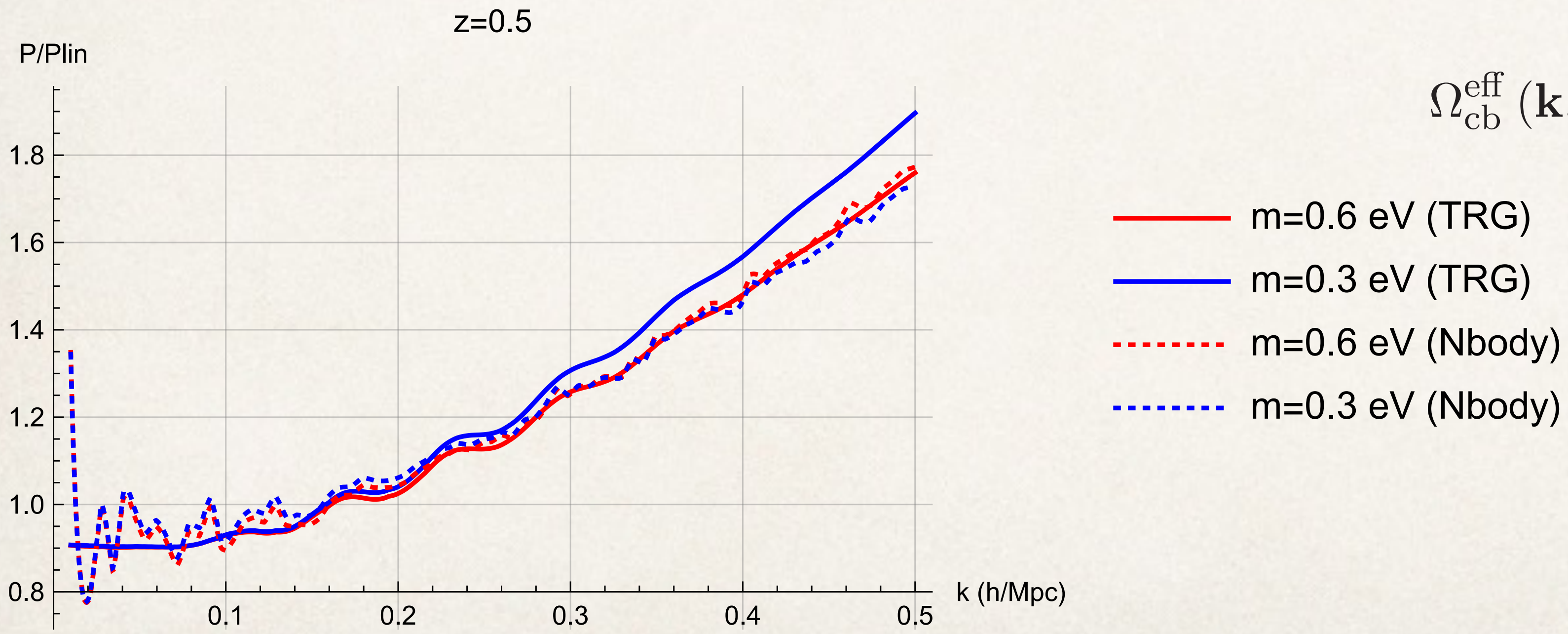
nonlinear contributions
to the PS at all SPT orders

differs from SPT
starting from 2-loops

Neutrinos with simplest truncation



$$\mathbf{\Theta}(\mathbf{k},\,\eta) = \begin{pmatrix} 1 & -1 \\ -\dfrac{3}{2}\Omega_{\rm cb}^{\rm eff}(\mathbf{k},\,\eta) & 2 + \dfrac{d\log\mathcal{H}}{d\eta} \end{pmatrix}$$

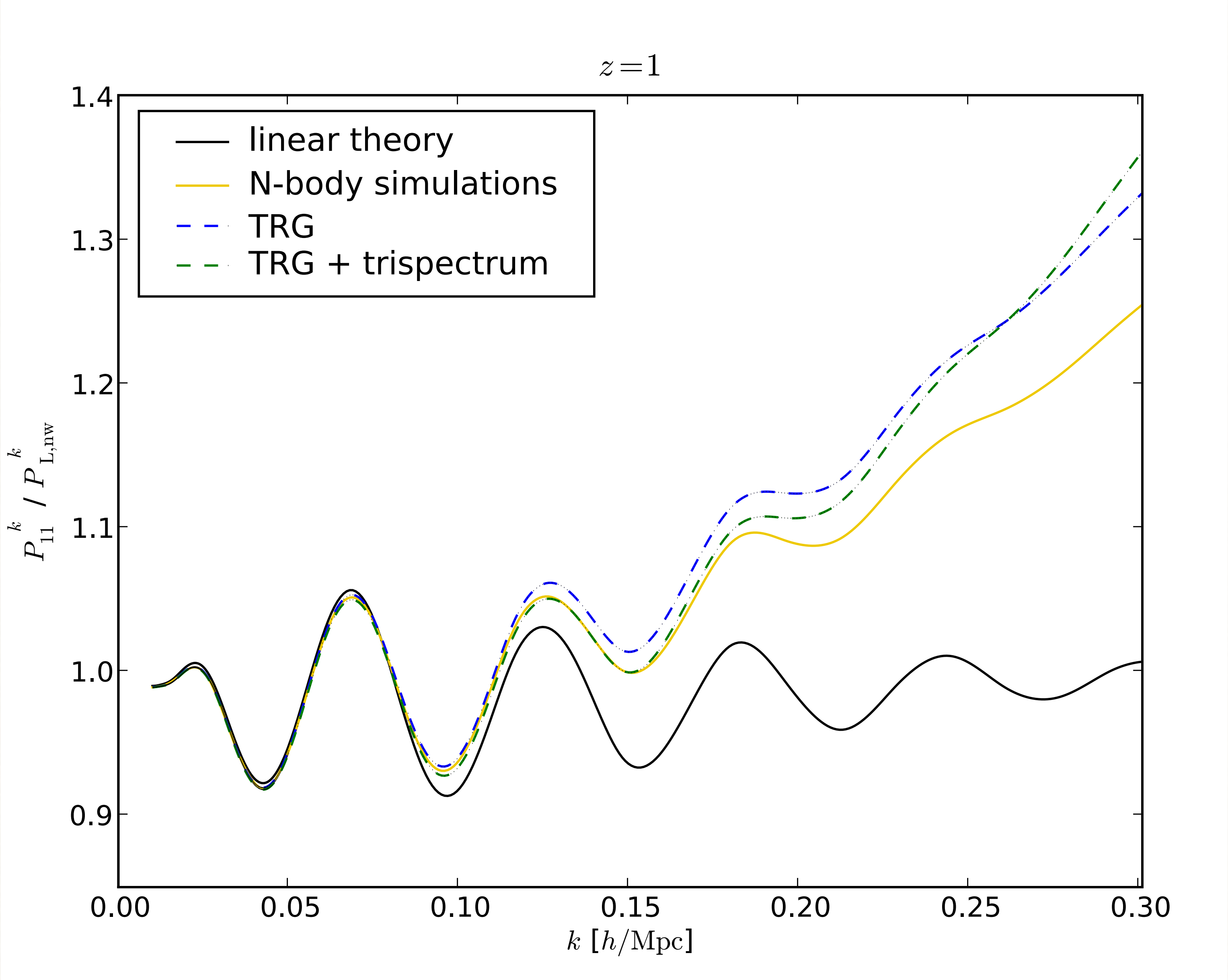


$$\Omega_{\rm cb}^{\rm eff}\,(\mathbf{k},\,\tau) \equiv \Omega_m\,(\tau)\,(1-f_\nu)\left(1 + \frac{f_\nu \delta_\nu^L(\mathbf{k},\,\tau)}{(1-f_\nu)\delta_{\rm cb}^L(\mathbf{k},\,\tau)}\right)$$

Belforti, Noda, MP, in progress

Improved momentum dependence

Next Truncation: including the trispectrum



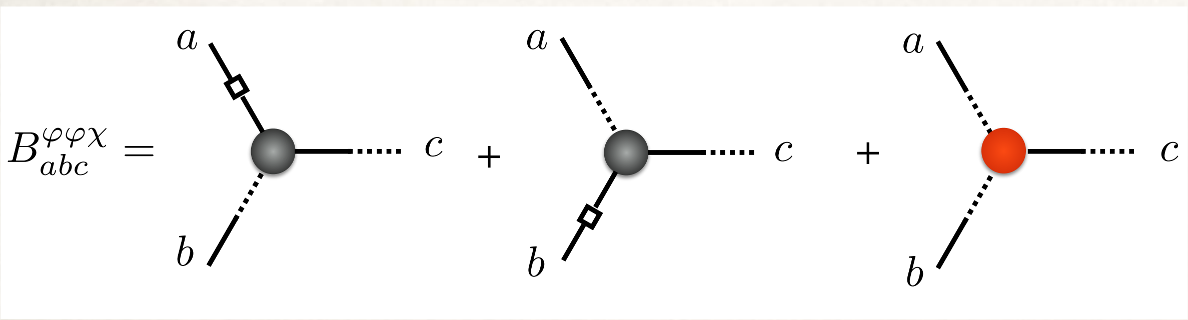
differs from SPT starting from 3-loops

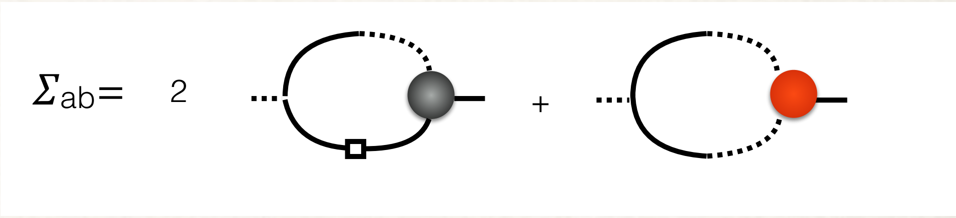
but still misses a crucial effect

Juergens, Bartelmann, 2012

Improving P^P

$$P^P_{ab}(k,\eta) = G_{ac}(k;\eta,\eta_{in})G_{bd}(k;\eta,\eta_{in})u_b u_d P^0(k)$$





$$\partial_\eta P^P_{ab}(k;\eta,\eta) = -\Omega_{ac}P^P_{cb}(k;\eta,\eta) - \Omega_{bc}P^P_{ac}(k;\eta,\eta)$$

Exact equation

$$+\int_{\eta_{in}} ds\ \left[\Sigma_{ac}(k;\eta,s)P^P_{cb}(k;s,\eta) + \Sigma_{bc}(k;\eta,s)P^P_{ac}(k;\eta,s)\right]$$

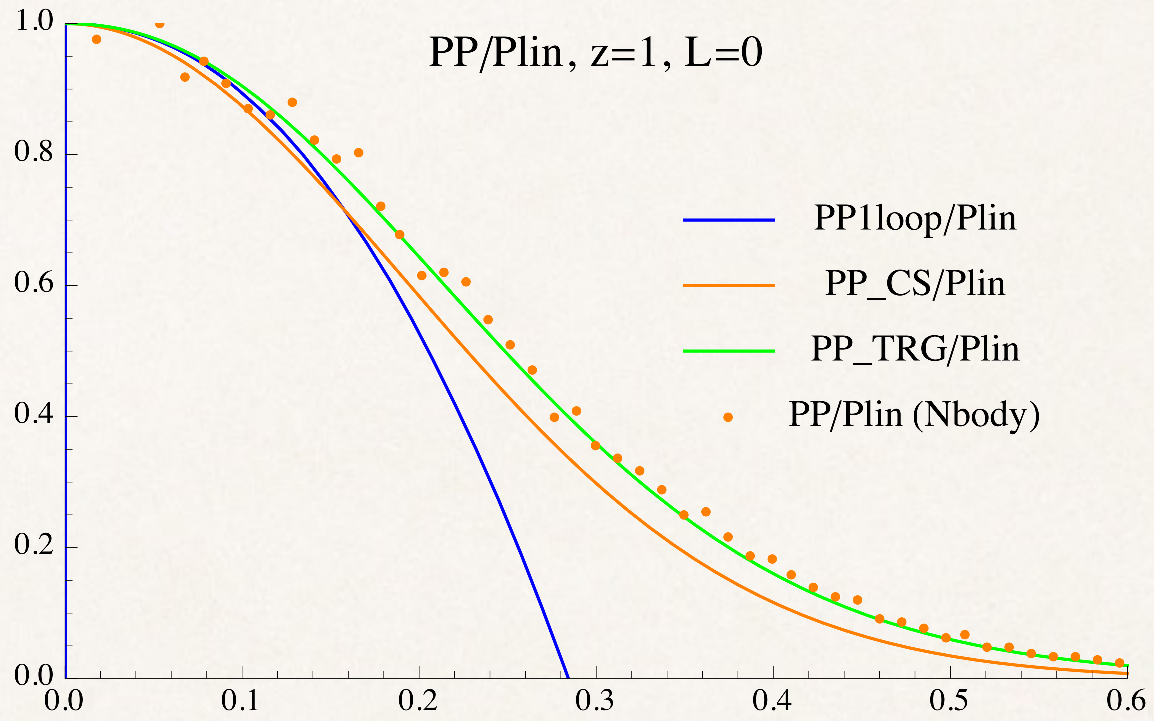
$\Sigma_{ab}(k;\eta,s) \rightarrow \Sigma^{1-loop}_{ab}(k;\eta,s)$ for $k \rightarrow 0$

$\Sigma_{ab}(k;\eta,s) \rightarrow -k^2\sigma_v^2(z)e^{\eta+s}g_{ab}(\eta;s)$ for $k \rightarrow \infty$ (exact factorization, exponential damping as in Crocce-Scoccimarro)

Anselmi, Matarrese, MP, 1011.4477

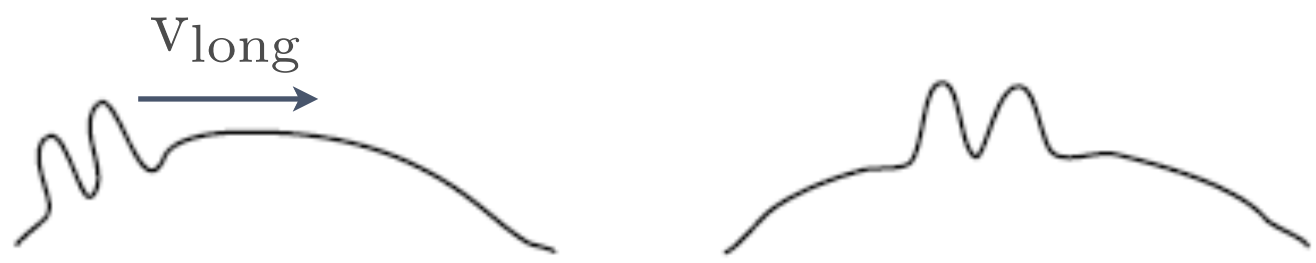
Peloso, MP, Viel, Villaescusa-Navarro, in preparation

Zel'dovich and beyond



Large k limit: Zel'dovich
Small k limit: 1-loop
Interpolation built in the equation!

How to include Bulk Motions



$$\bar{\delta}_\alpha(\mathbf{x},\tau) = \delta_\alpha(\mathbf{x} - \mathbf{D}_\alpha(\mathbf{x},\tau),\tau)$$

$$\mathbf{D}_\alpha(\mathbf{x},\tau) \equiv \int_{\tau_{in}}^{\tau} d\tau' \mathbf{v}_{\alpha,\mathrm{long}}(\mathbf{x},\tau') \simeq \mathbf{D}_\alpha(\tau)$$

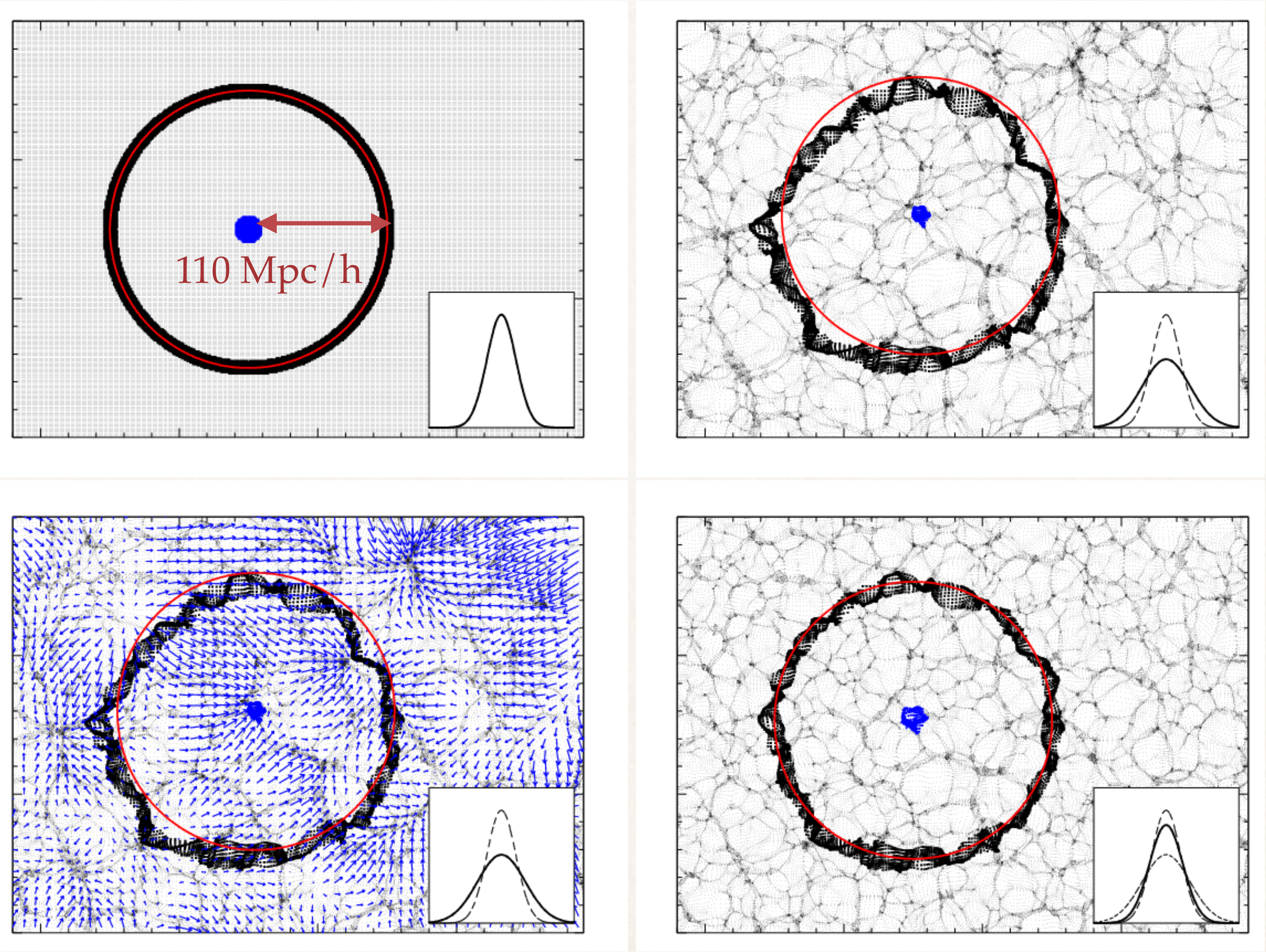
$$\langle \delta_\alpha(\mathbf{k},\tau)\delta_\alpha(\mathbf{k}',\tau')\rangle = \langle \bar{\delta}_\alpha(\mathbf{k},\tau)\bar{\delta}_\alpha(\mathbf{k}',\tau')\rangle \langle e^{-i\mathbf{k}\cdot(\mathbf{D}_\alpha(\tau)-\mathbf{D}_\alpha(\tau'))}\rangle$$

$$= \langle \bar{\delta}_\alpha(\mathbf{k},\tau)\bar{\delta}_\alpha(\mathbf{k}',\tau')\rangle e^{\frac{-k^2\sigma_v^2(D(\tau)-D(\tau'))^2}{2}}$$

$$\sigma_v^2 = -\frac{1}{3\mathcal{H}^2 f^2}\int^\Lambda d^3q \langle v^i_{long}(q) v^i_{long}(q)\rangle' = \frac{1}{3}\int^\Lambda d^3q \frac{P^0(q)}{q^2}$$

Resummations (~Zel'dovich)
take into account the large scale bulk motions

Large scale flows and BAO's



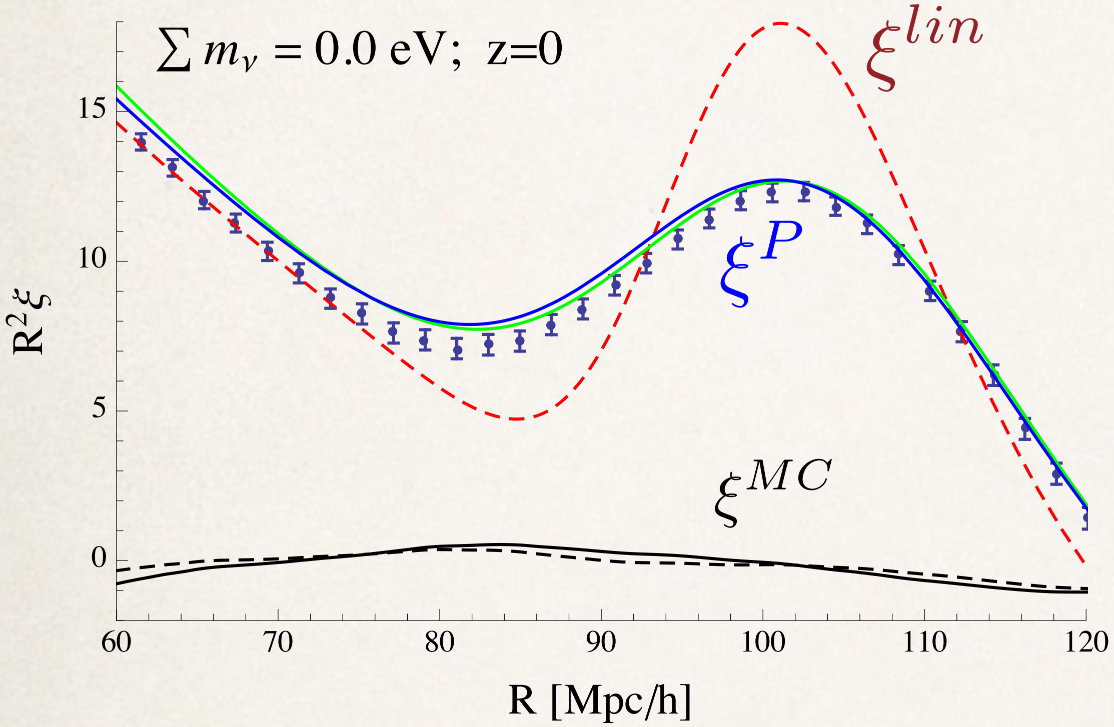
O(10 Mpc)

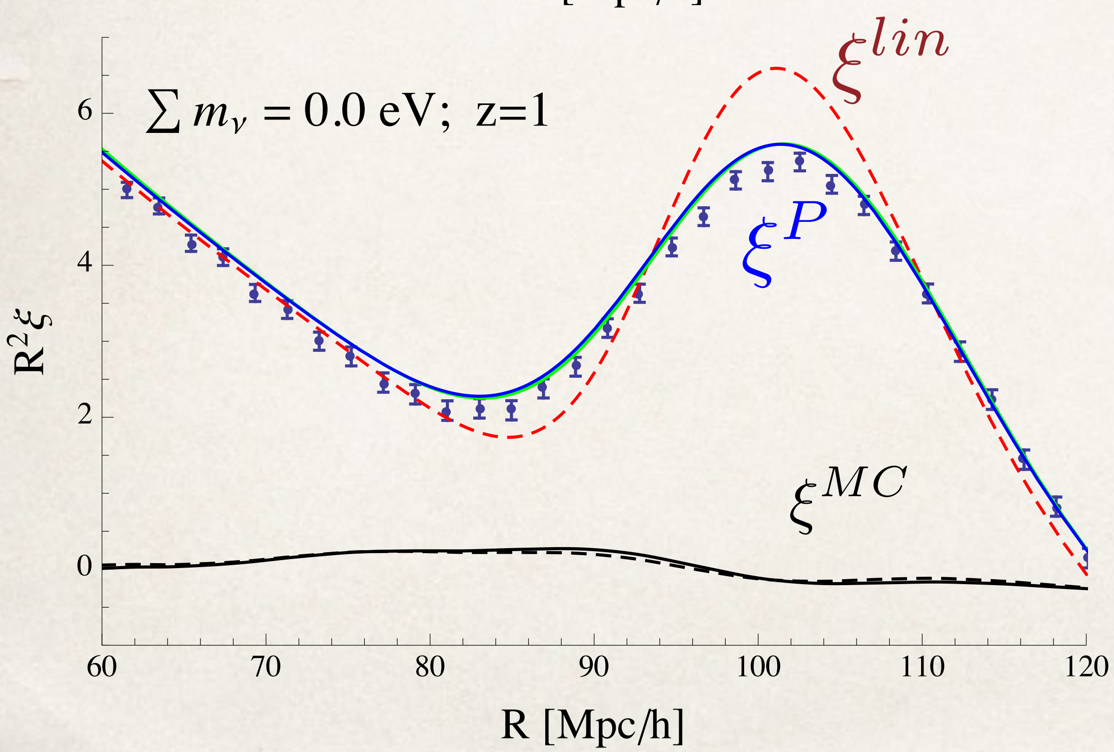
displacements

reconstruction

Seo et al, 0910.5005, Padmanabhan et al 1202.0090, Tassev, Zaldarriaga 1203.6066, …

Effect on the Correlation Function





Most of the information on the BAO peak is contained in the propagator part

The widening of the peak can be reproduced by Zel'dovich approximation (and improvements of it)

The widening of the peak contains robust physical information (not a parameter to marginalize!)

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

(simplified) Zel'dovich approximation

$$G^{Zeld}(k,z) = \mathrm{e}^{-\frac{k^2\sigma_v^2(z)}{2}}$$

$$P_{11}^{P}(k,z) = e^{-\frac{k^2\sigma_v^2(z)}{2}} P^{lin}(k;z)$$

$$\sigma_v^2(z) = \frac{1}{3}\int \frac{d^3q}{(2\pi)^3}\frac{P^{lin}(q,z)}{q^2}$$

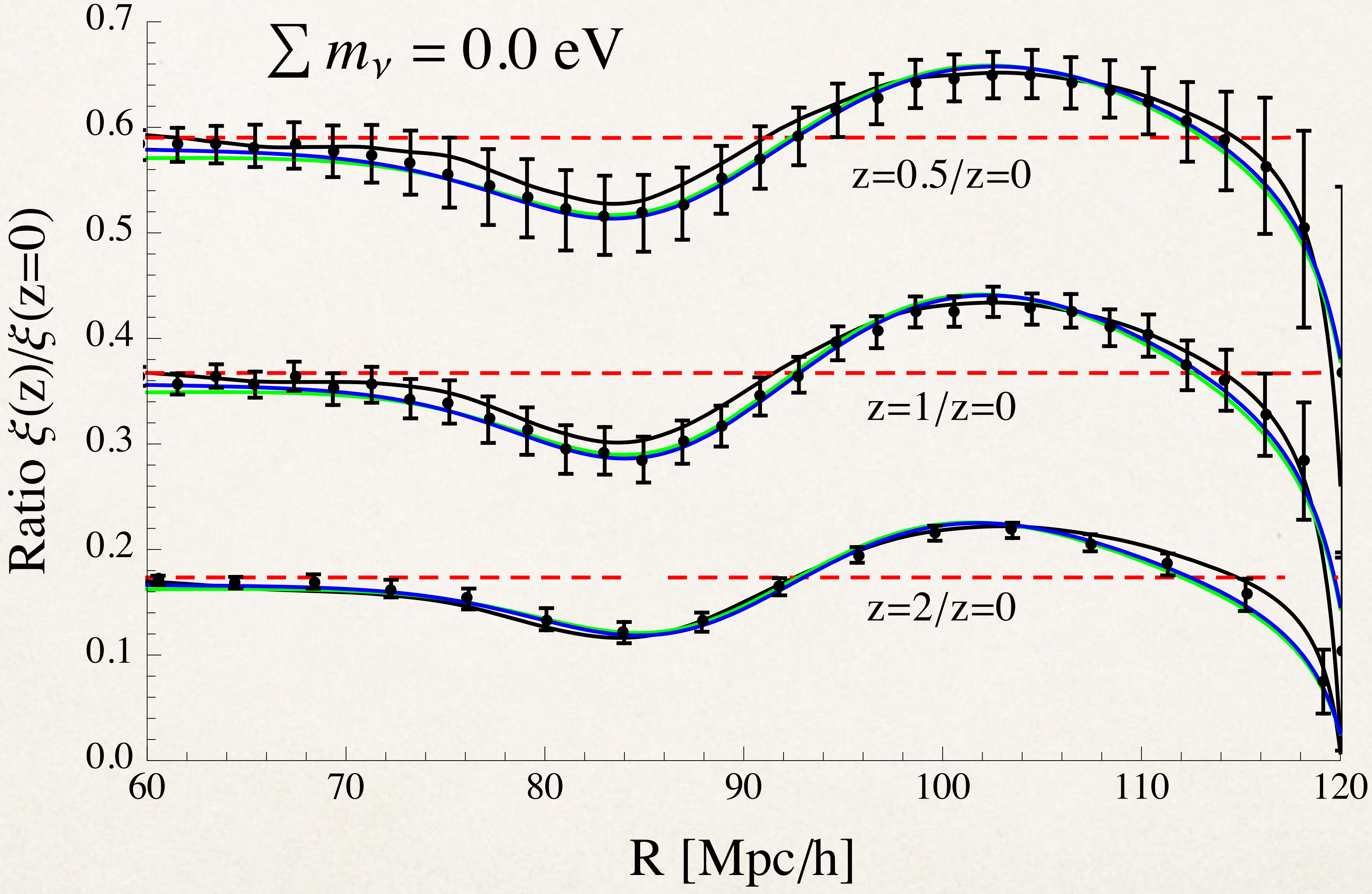
linear velocity dispersion:
contains information on linear PS, growth factor,…

$$\delta\xi(R) = \frac{1}{2\pi^2}\int dq\, q^2\, \delta P^{lin}(q)\left(\frac{\sin(qR)}{qR}\, e^{-q^2\sigma_v^2} - \frac{1}{3}\frac{\xi_2(R)}{q^2R^2}\right)$$

$$\xi_n(R) \equiv \frac{1}{2\pi^2 R}\int_0^\infty dq\, q\,(qR)^n \sin(qR)\, P(q)$$

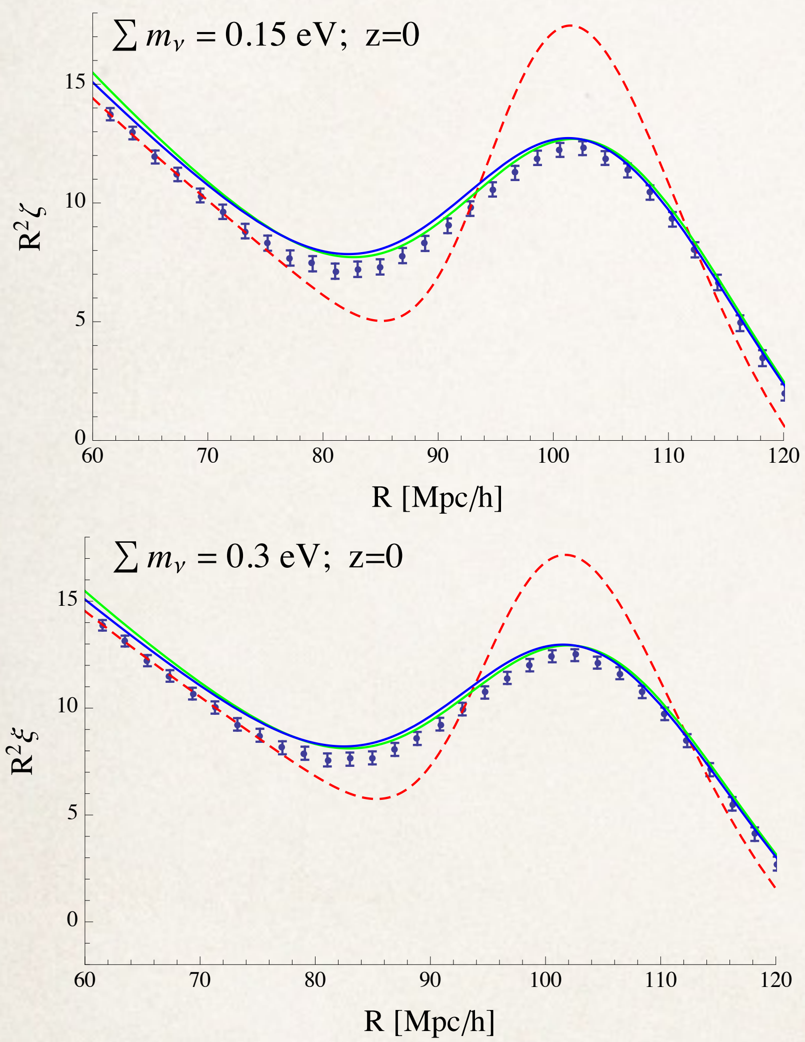
Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

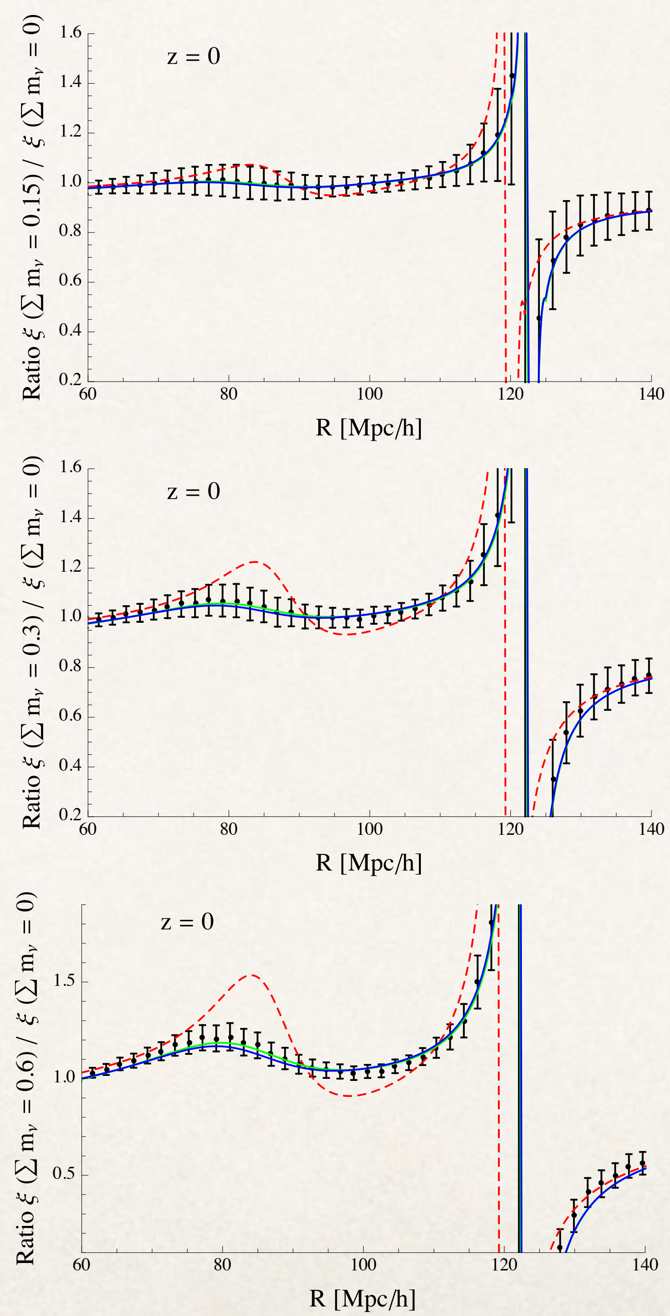
Redshift ratios



Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

Effect of Massive neutrinos on BAO peak





$$P_{11}^{P}(k,z) = e^{-\frac{k^2\sigma_v^2(z)}{2}} P^{lin}(k;z)$$

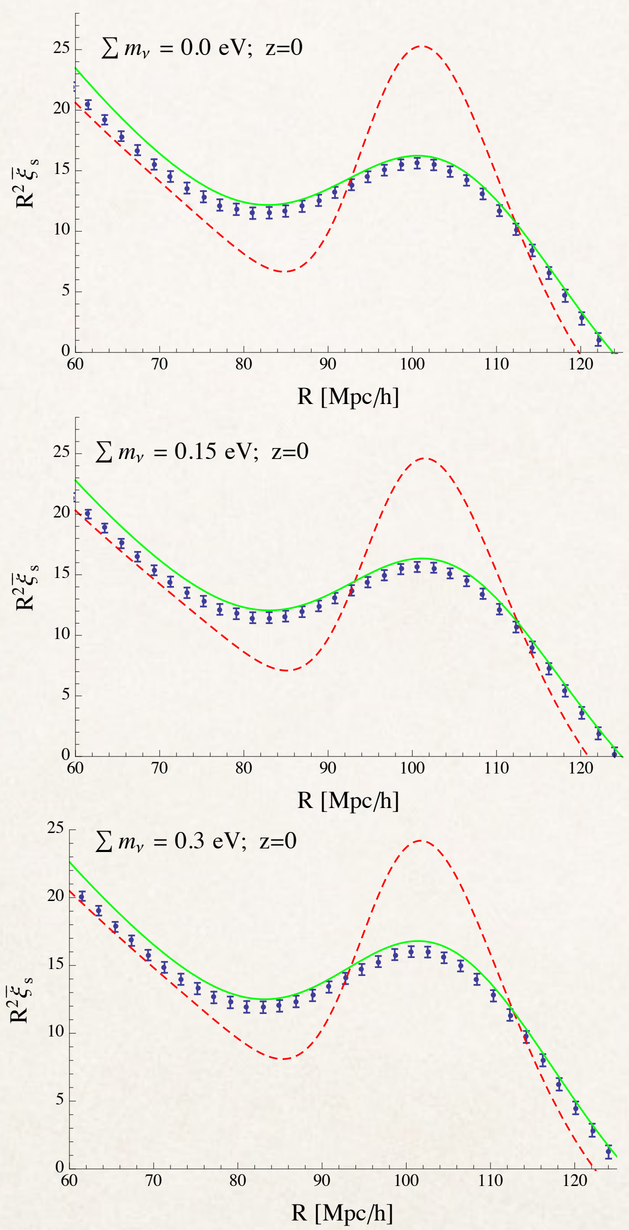
increasing neutrino masses,
Plin decreases, but also velocity dispersion decreases.

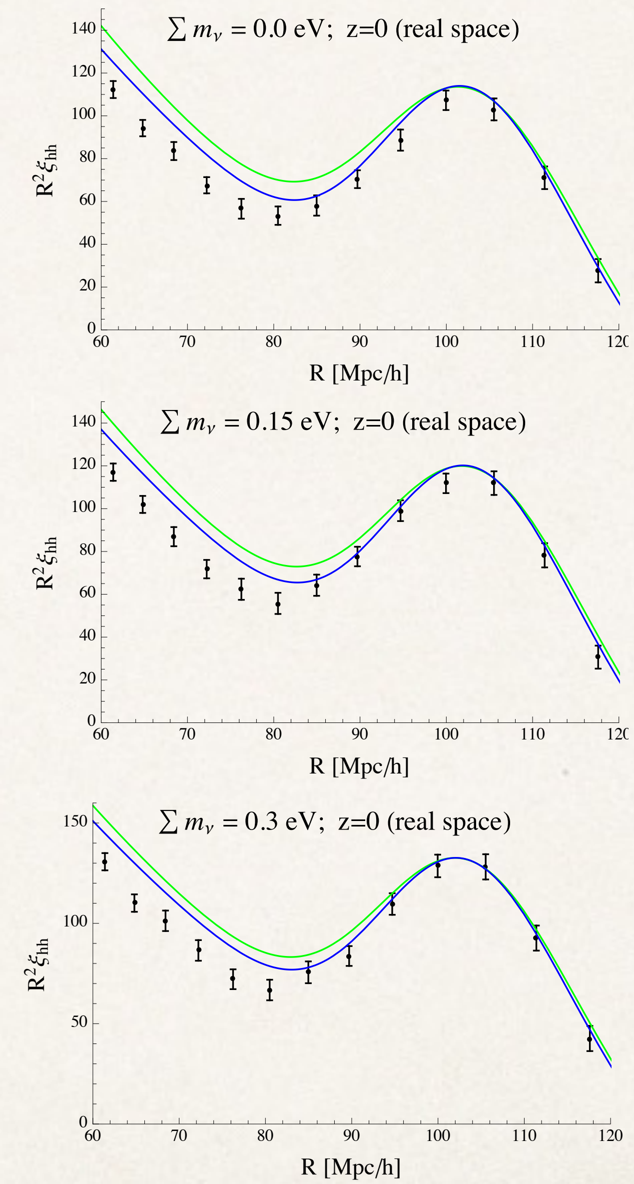
$$\sum m_\nu = 0.15 \quad \text{eV} \qquad \downarrow 0.6\%$$

$$\sum m_\nu = 0.3 \quad \text{eV} \qquad \uparrow 1.2\%$$

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

Massive neutrinos





Redshift space

Halos

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

Dealing with the MC

$$P_{ab}(k;\eta,\eta') = G_{ac}(k;\eta,\eta_{in})G_{bd}(k;\eta',\eta_{in})P^0(k)u_c u_d +$$
$$\int_{\eta_{in}} ds\, ds'\; G_{ac}(k;\eta,s)G_{bd}(k;\eta',s')\Phi_{cd}(k;s,s')$$
$$\equiv P^P_{ab}(k;\eta,\eta') + P^{MC}_{ab}(k;\eta,\eta')\,.$$

Putting everything together: TRG with IR resummation and UV sources

$$\partial_\eta P_{ab}^{MC}(k;\eta,\eta) = -\Omega_{ac}P_{cb}^{MC}(k;\eta)$$ linear growth

$$+\int^{\eta} ds\, \Sigma_{ac}(k;\eta,s)P_{cb}^{MC}(k;s,\eta)$$ IR (propagator) effects

$$+e^{\eta}\int d^3q\gamma_{acd}(k,q)B_{cdb}^{MC}(q,k;\eta)$$ Intermediate scales: (resummed) SPT

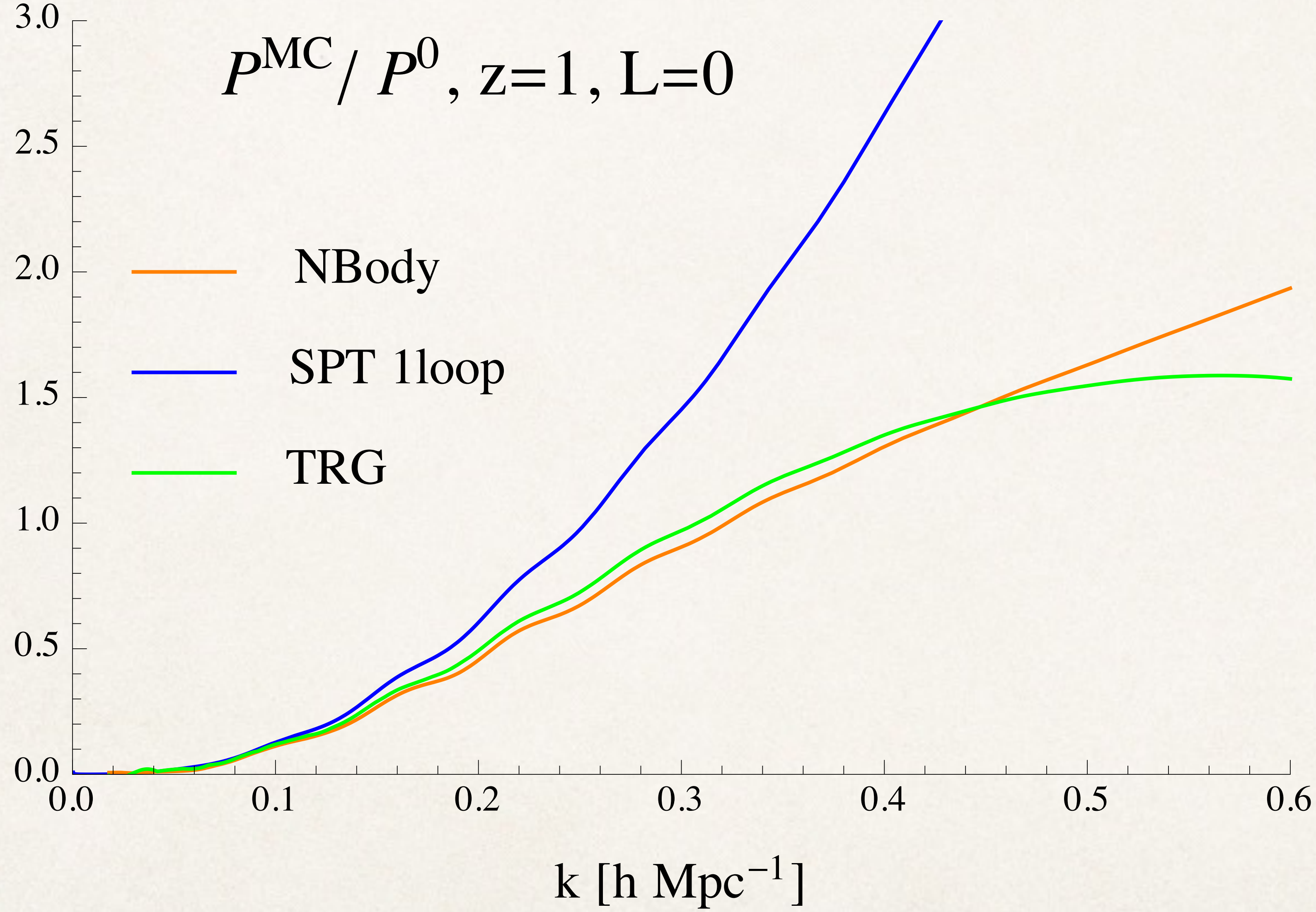
$$-\langle h_a(\mathbf{k},\eta)\varphi_b^{MC}(-\mathbf{k},\eta)\rangle$$ UV sources (from Nbody)

$$+(a\leftrightarrow b)$$

Improved TRG

Peloso, MP, Viel, Villaescusa-Navarro, in preparation

IR resummation for P^{MC}



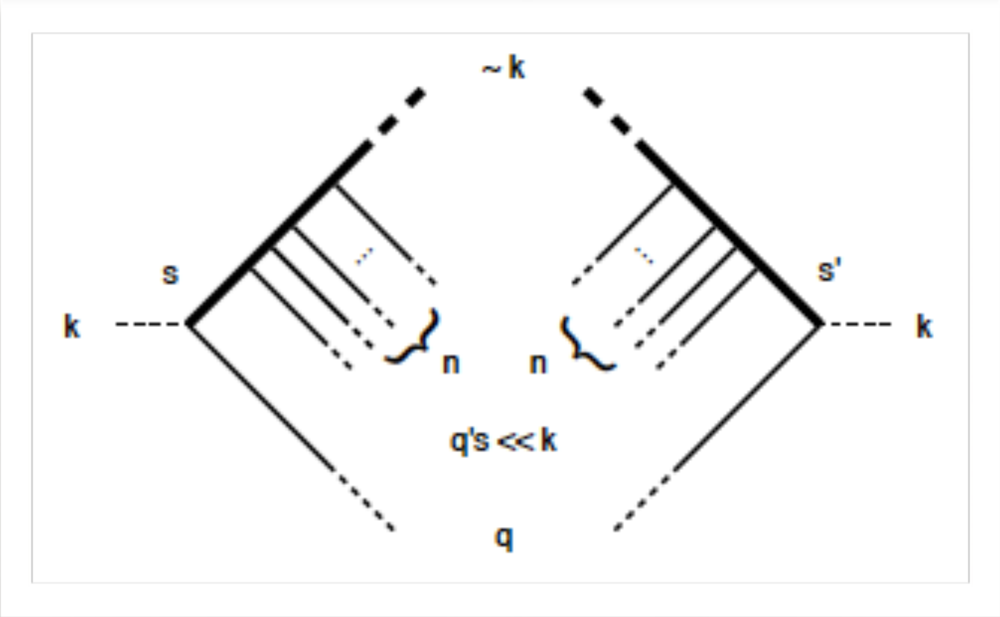
Sigma included, Phi @ 1-loop, no UV sources

No hidden parameters. As fast as a 1-loop SPT computation

Beyond 1-loop $\Phi_{ab}(k; s, s')$

Anselmi, MP, 1205.2235

large k



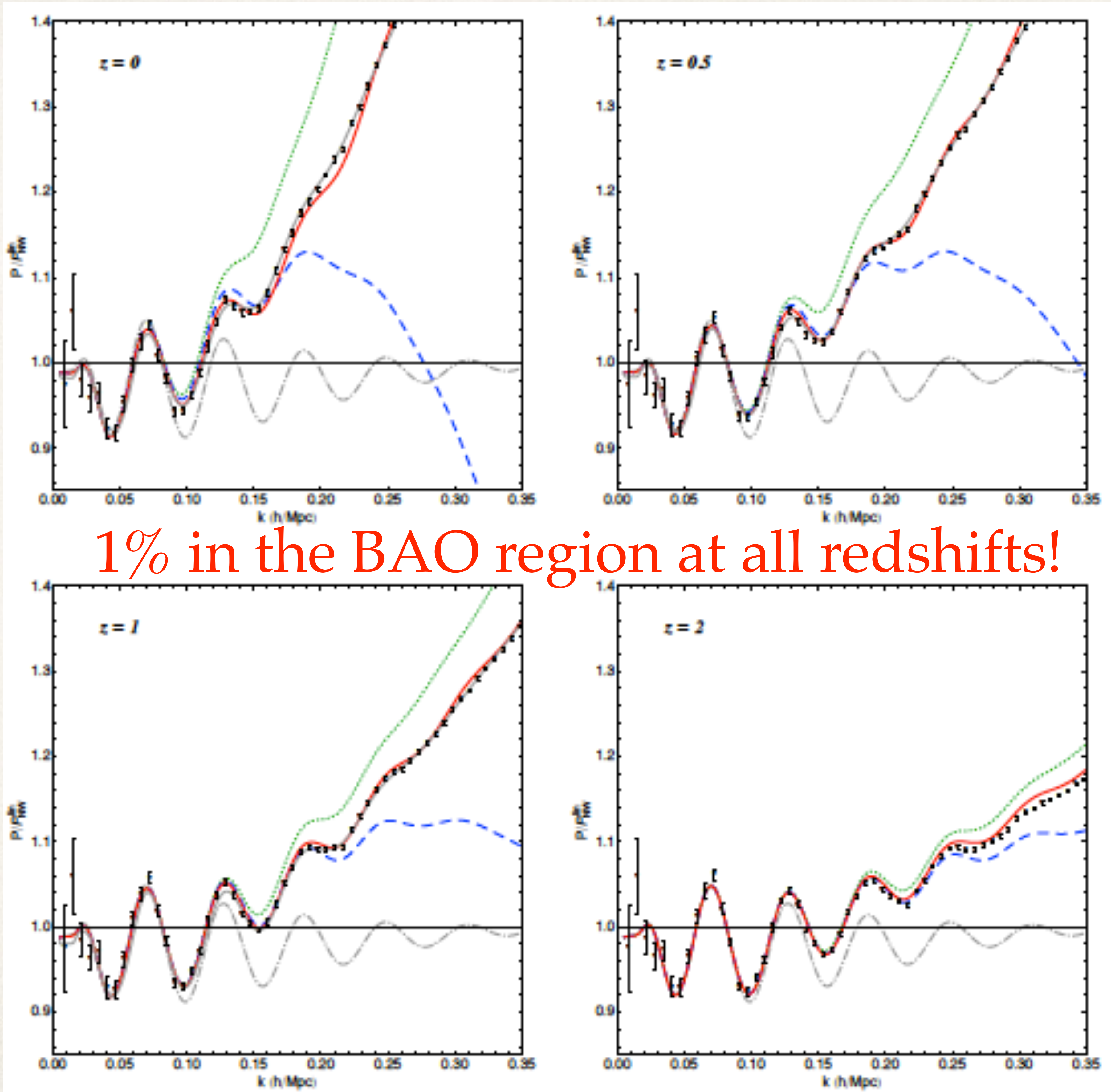
leading contributions to Phi:

1 "hard" loop momentum, n-1 "soft" ones

$$\tilde{\Phi}_{ab}(k; s, s') \to e^{-\frac{k^2\sigma_v^2}{2}(e^s - e^{s'})^2} \left[\Phi_{ab}^{(1)}(k; s, s') + \left(k^2\sigma_v^2\, e^{s+s'}\right)^2 P(k) u_a u_b\right]$$

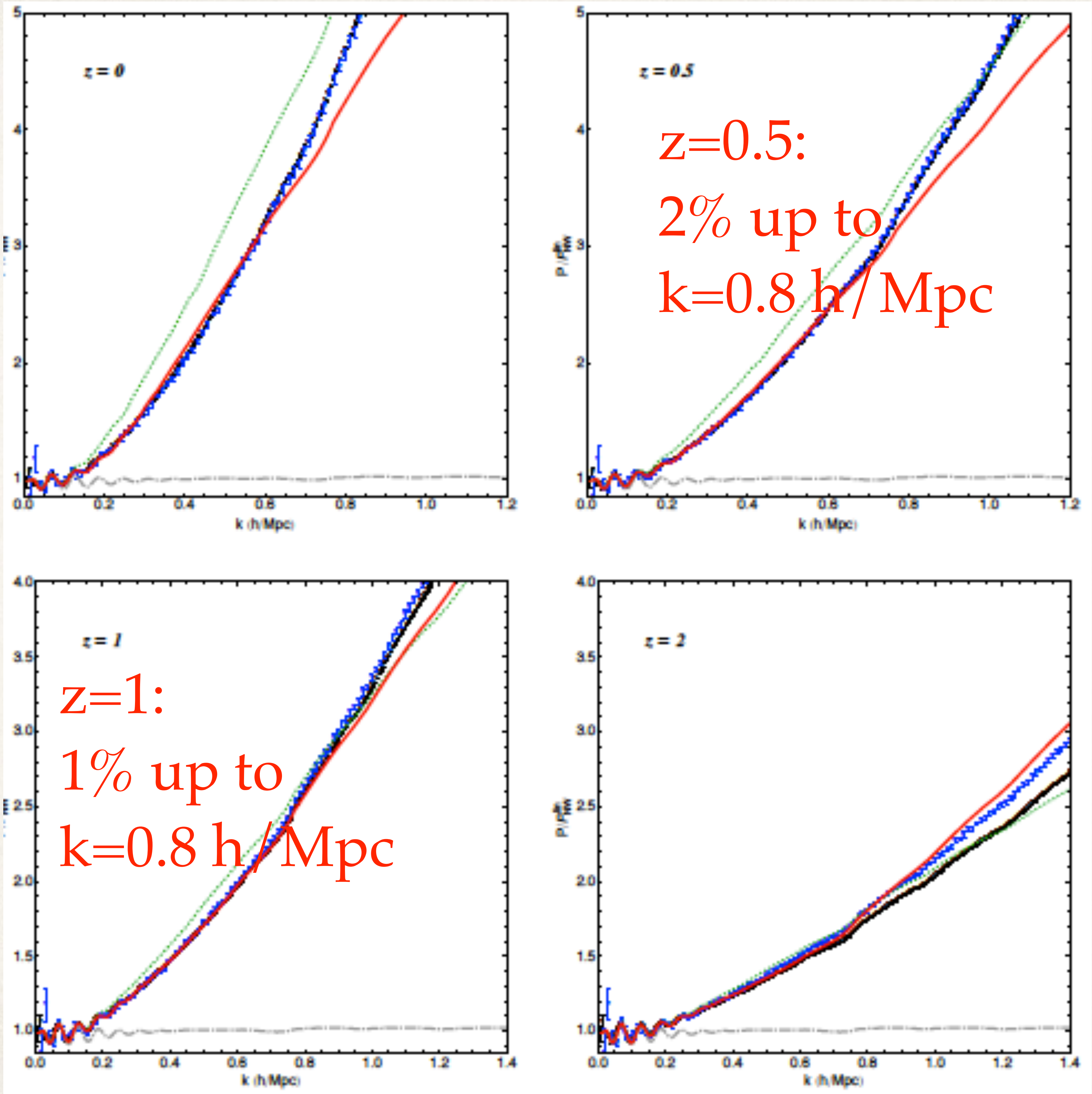
Can be obtained in eRPT:
tree-level=UV limit

BAO scales

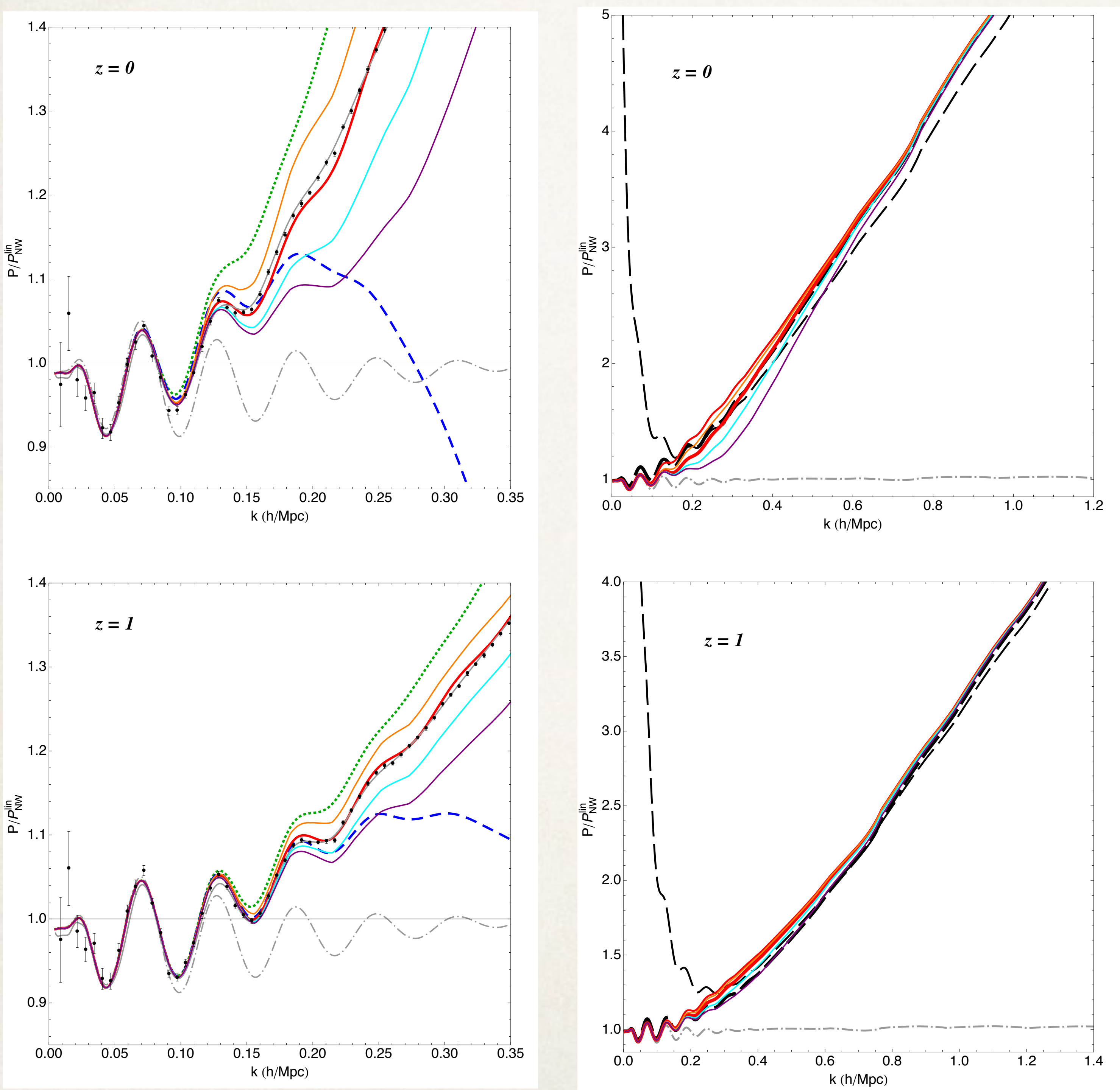


1% in the BAO region at all redshifts!

large k



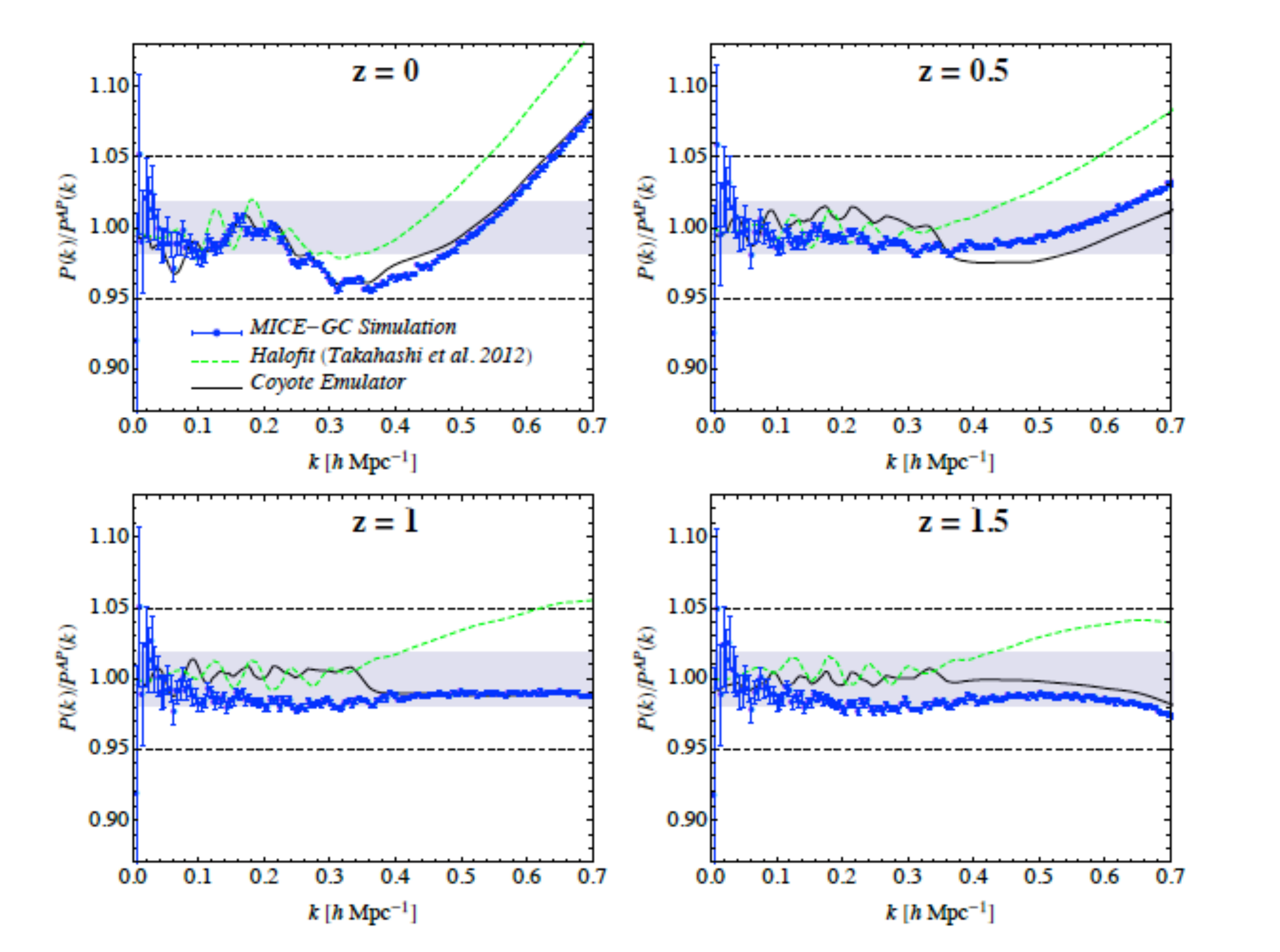
One parameter



$$\tilde{\Phi} G^{A}_{ab}(k;\eta) + \frac{\left(\frac{k}{\bar{\bar{k}}}\right)^4}{1+\left(\frac{k}{\bar{\bar{k}}}\right)^4} \tilde{\Phi} G^{B}_{ab}(k;\eta)$$

"1loop" "higher orders"

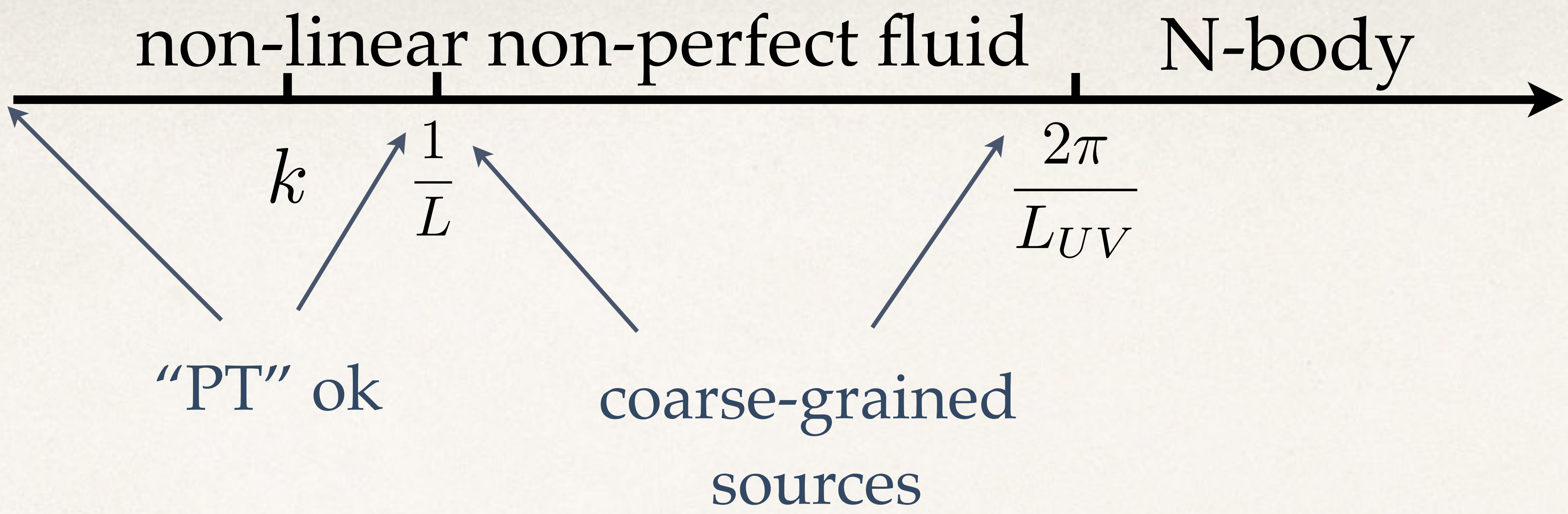
with resummations of the MC part



Anselmi, Lopez-Nacir, Sefusatti, 2014

Dealing with the UV

- General idea: take the UV physics from N-body simulations and use (resummed) PT only for the large and intermediate scales



Physics at k must be independent on L, L_uv
("Wilsonian approach")

Expansion in sources:

$$\langle \delta\delta \rangle_J = \langle \delta\delta \rangle_{J=0} + \langle \delta J \delta \rangle_{J=0} + \frac{1}{2} \langle \delta J J \delta \rangle_{J=0} + \cdots$$

computed in PT
with cutoff at 1/L

measured from
simulations

Vlasov Equation

Liouville theorem+ neglect non-gravitational interactions:

$$\frac{d}{d\tau} f_{mic} = \left[\frac{\partial}{\partial\tau} + \frac{p^i}{am}\frac{\partial}{\partial x^i} - am\nabla_x^i \phi(\mathbf{x},\tau)\right] f_{mic}(\mathbf{x},\mathbf{p},\tau) = 0$$

moments:

$$n_{mic}(\mathbf{x},\tau) = \int d^3p f_{mic}(\mathbf{x},\mathbf{p},\tau)$$ density

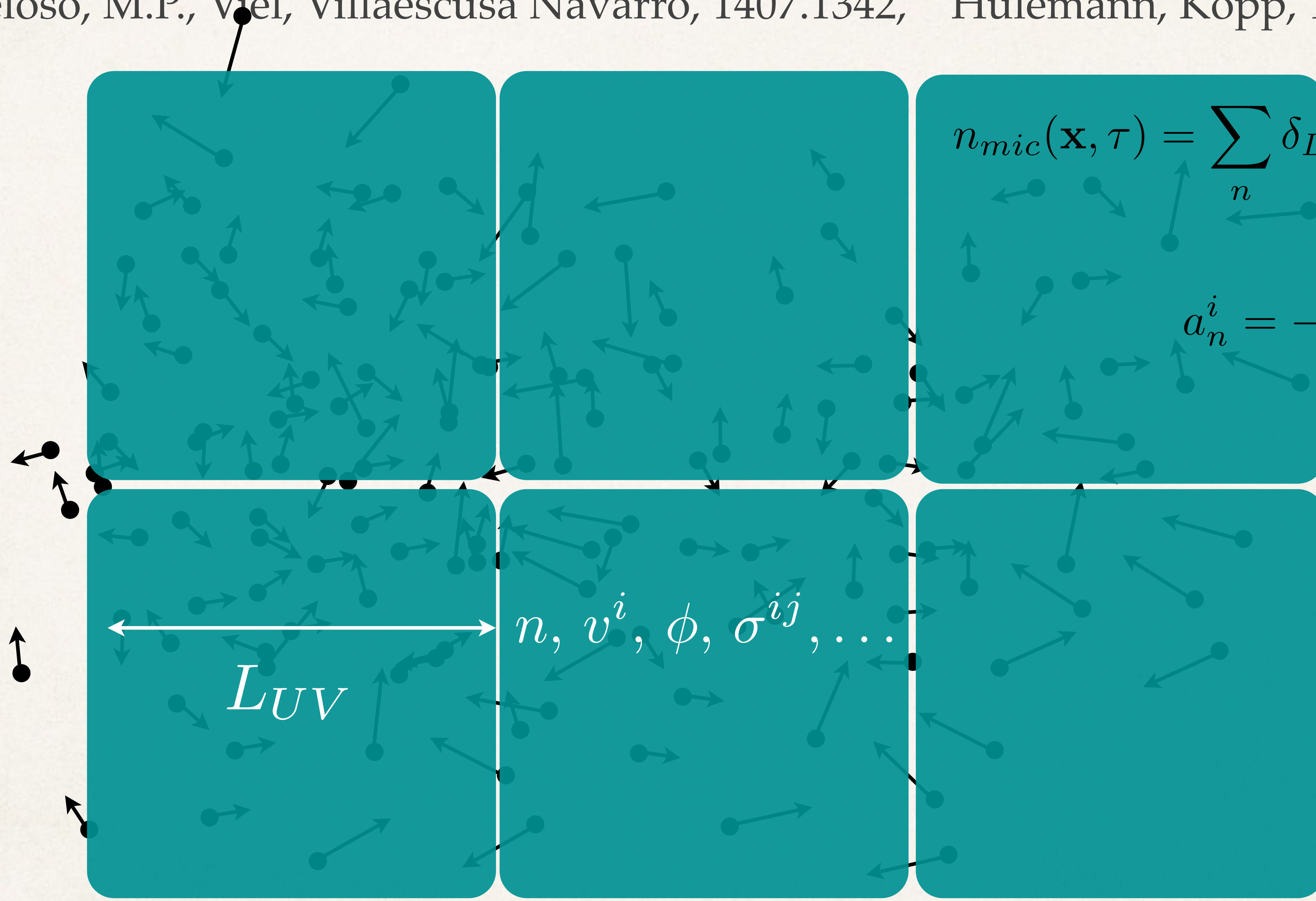
$$\mathbf{v}_{mic}(\mathbf{x},\tau) = \frac{1}{n_{mic}(\mathbf{x},\tau)} \int d^3p \, \frac{\mathbf{p}}{am} f_{mic}(\mathbf{x},\mathbf{p},\tau)$$ velocity

$$\sigma^{ij}_{mic}(\mathbf{x},\tau) = \frac{1}{n_{mic}(\mathbf{x},\tau)} \int d^3p \, \frac{p^i}{am}\frac{p^j}{am} f_{mic}(\mathbf{x},\mathbf{p},\tau) - v^i_{mic}(\mathbf{x},\tau) v^j_{mic}(\mathbf{x},\tau)$$ velocity dispersion

…

From particles to fluids

Buchert, Dominguez, '05, Pueblas Scoccimarro, '09, Baumann et al. '10
M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203, Carrasco, Hertzberg, Senatore,1206.2976 .
Manzotti, Peloso, M.P., Viel, Villaescusa Navarro, 1407.1342, Hulemann, Kopp, 1407.4810 …



$$n_{mic}(\mathbf{x},\tau) = \sum_n \delta_D(\mathbf{x} - \mathbf{x}_n(\tau))\,,$$

$$v_n^i = \dot{x}_n(\tau)\,,$$

$$a_n^i = -\nabla_x^i \phi_{mic}(\mathbf{x},\tau)$$

$$f_{mic}(x,p,\tau) = \sum_n \delta_D(x - x_n(\tau))\,\delta_D(p - p_n(\tau))$$

$$f(x,p,\tau) = \frac{1}{V}\int d^3y\, W(y/L_{UV})\, f_{mic}(x+y,p,\tau)$$

Satisfies the "Vlasov eq."

Coarse-grained Vlasov equation

large scales

$$\left[\frac{\partial}{\partial \tau} + \frac{p^i}{am}\frac{\partial}{\partial x^i} - am\nabla_x^i \phi(\mathbf{x},\tau)\frac{\partial}{\partial p^i}\right] f(\mathbf{x},\mathbf{p},\tau) =$$

$$am\left[\langle \frac{\partial}{\partial p^i} f_{mic} \nabla^i \phi_{mic}\rangle_{L_{UV}}(\mathbf{x},\mathbf{p},\tau) - \frac{\partial}{\partial p^i} f(\mathbf{x},\mathbf{p},\tau)\nabla_x^i \phi(\mathbf{x},\tau)\right]$$

short scales

$$\langle g \rangle_{L_{UV}}(\mathbf{x}) \equiv \frac{1}{V_{UV}} \int d^3y\, \mathcal{W}(y/L_{UV}) g(\mathbf{x}+\mathbf{y})$$

$$\phi = \langle \phi_{mic} \rangle_{L_{UV}}$$

$$f = \langle f_{mic} \rangle_{L_{UV}}$$

Vlasov equation in the L_uv ➞ 0 limit!

Taking moments…

Exact large scale dynamics for density and velocity fields

$$\frac{\partial}{\partial \tau}\delta(\mathbf{x}) + \frac{\partial}{\partial x^i}\left[(1+\delta(\mathbf{x}))v^i(\mathbf{x})\right] = 0$$

$$\frac{\partial}{\partial \tau}v^i(\mathbf{x}) + \mathcal{H}v^i(\mathbf{x}) + v^k(\mathbf{x})\frac{\partial}{\partial x^k}v^i(\mathbf{x}) = -\nabla_x^i \phi(\mathbf{x}) - \underline{J_\sigma^i(\mathbf{x}) - J_1^i(\mathbf{x})}$$

$$\nabla^2\phi(\mathbf{x}) = \frac{3}{2}\Omega_M \mathcal{H}^2 \delta(\mathbf{x})$$

$$n(\mathbf{x}) = n_0(1+\delta(\mathbf{x})) = n_0(1+\langle\delta_{mic}\rangle(\mathbf{x}))$$

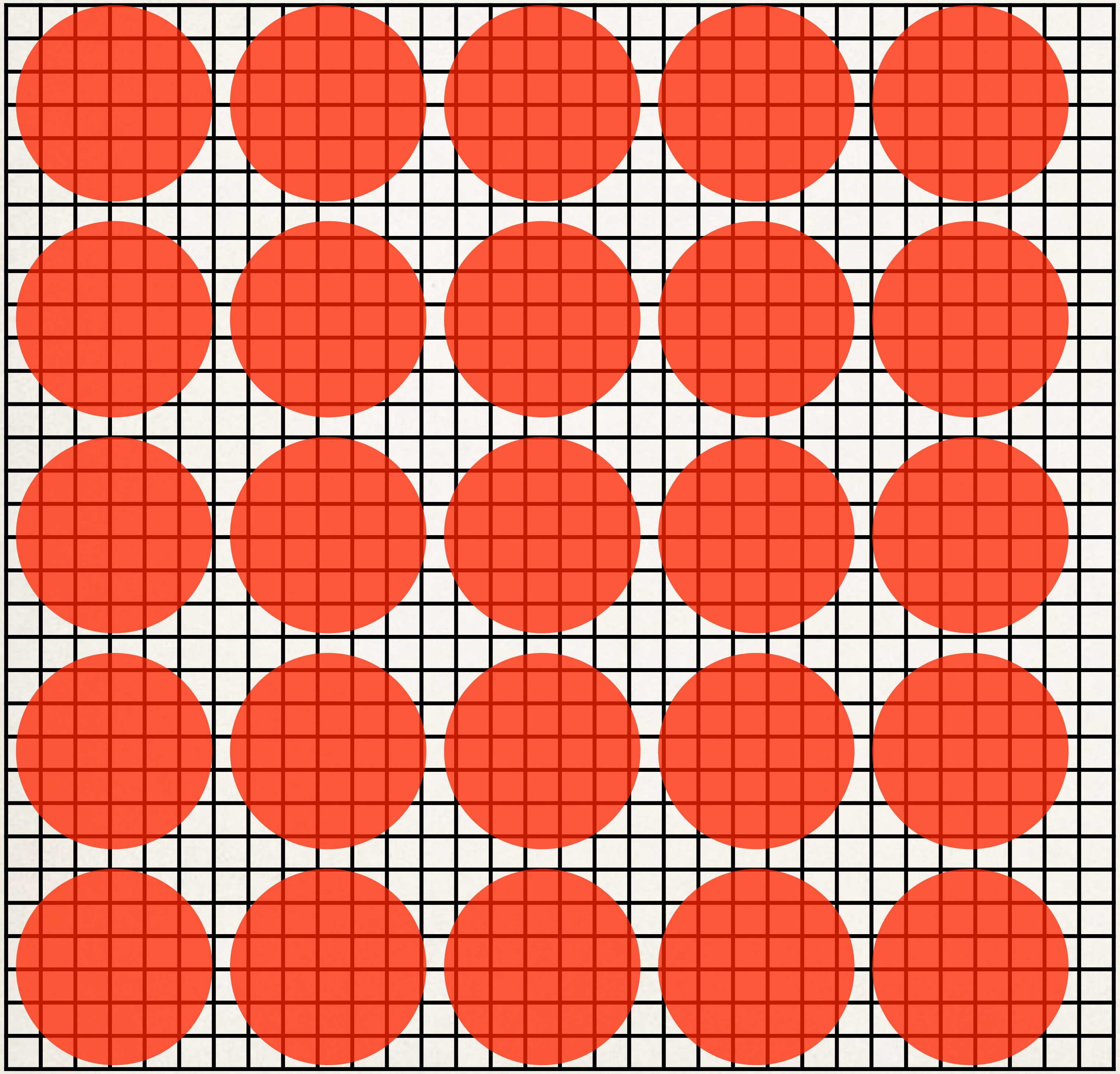
$$v^i(\mathbf{x}) = \frac{\langle(1+\delta_{mic})v^i_{mic}\rangle(\mathbf{x})}{1+\delta(\mathbf{x})}$$

external input on UV-physics needed

$$\left\{\begin{array}{l} J_\sigma^i(\mathbf{x}) \equiv \dfrac{1}{n(\mathbf{x})}\dfrac{\partial}{\partial x^k}(n(\mathbf{x})\sigma^{ki}(\mathbf{x})) \\[2ex] J_1^i(\mathbf{x}) \equiv \dfrac{1}{n(\mathbf{x})}\left(\langle n_{mic}\nabla^i\phi_{mic}\rangle(\mathbf{x}) - n(\mathbf{x})\nabla^i\phi(\mathbf{x})\right) \end{array}\right.$$

Measuring the sources in Nbody simulation

Manzotti, Peloso, MP,
Villaescusa-Navarro, Viel, 1407.1342



$$L_{UV} = 1,\ 2,\ 4\ \mathrm{Mpc/h}$$

$$L_{UV}:\ \delta,\ v^i,\ J_1^i\,,\ J_\sigma^i$$

$$L:\ \bar{\delta},\ \bar{v}^i,\ \bar{J}_1^i,\ \bar{J}_\sigma^i$$

$$\mathcal{W}(R/L) = \left(\frac{2}{\pi}\right)^{3/2} \frac{1}{L^3}\, e^{-\frac{R^2}{2L^2}}$$

$$L_{box} = 512\,\mathrm{Mpc/h} \qquad N_{particles} = (512)^3$$

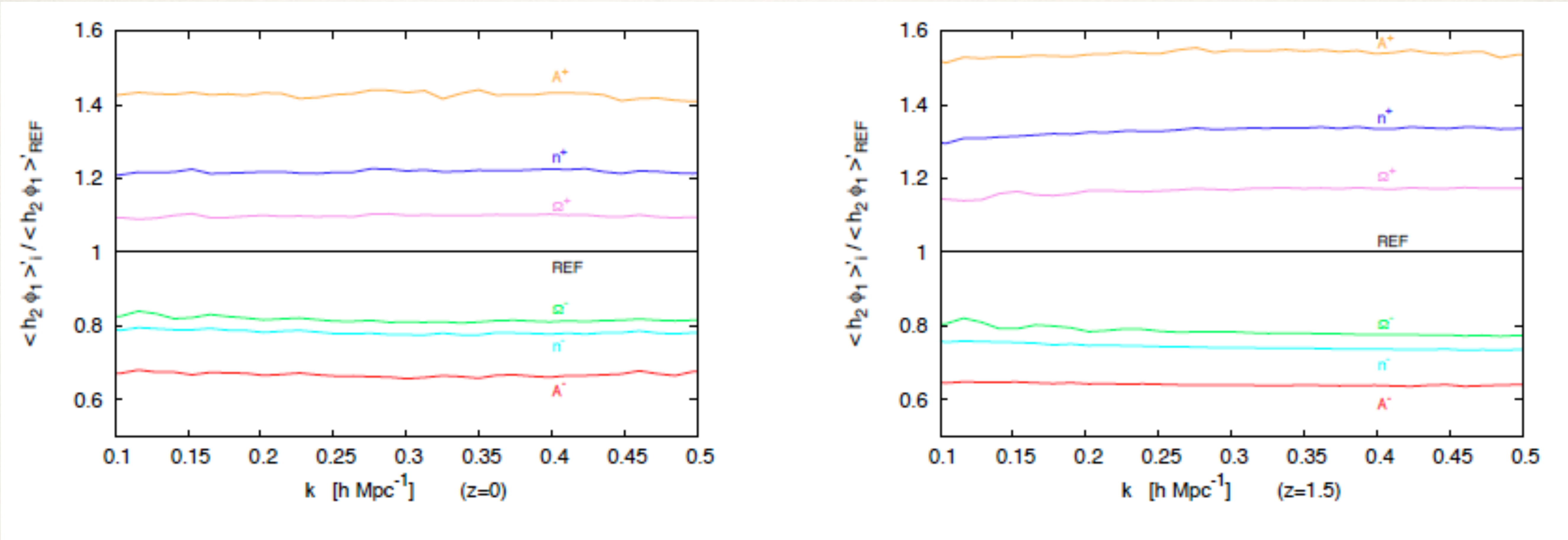
COSMOLOGY DEPENDENCE

Simulation Suite

Name	Ω_m	Ω_b	Ω_Λ	h	n_s	$A_s\,[10^{-9}]$
REF	0.271	0.045	0.729	0.703	0.966	2.42
A_s^-	0.271	0.045	0.729	0.703	0.966	1.95
A_s^+	0.271	0.045	0.729	0.703	0.966	3.0
n_s^-	0.271	0.045	0.729	0.703	0.932	2.42
n_s^+	0.271	0.045	0.729	0.703	1.000	2.42
Ω_m^-	0.247	0.045	0.753	0.703	0.966	2.42
Ω_m^+	0.289	0.045	0.711	0.703	0.966	2.42

$L_{box} = 512\,\mathrm{Mpc/h}$ $\qquad$ $N_{particles} = (512)^3$

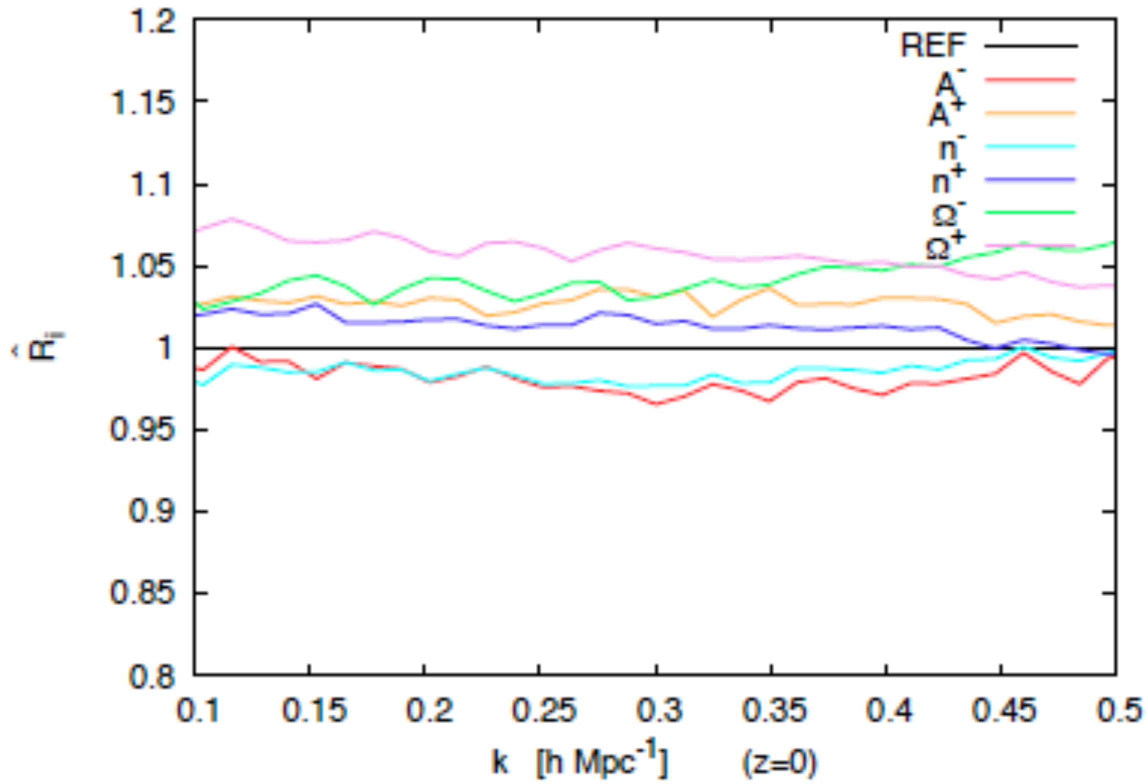
Ratios of UV source correlators



$$\frac{\langle J\delta\rangle_i}{\langle J\delta\rangle_{REF}}$$ From N-body

Scale-independent!!

Rescale using PT information



Amplitude rescaling captured by PT!!

Putting everything together: TRG with IR resummation and UV sources

$$\begin{aligned}\partial_\eta P_{ab}^{MC}(k;\eta,\eta) &= -\Omega_{ac}P_{cb}^{MC}(k;\eta) && \text{linear growth}\\
&+\int^\eta ds\,\Sigma_{ac}(k;\eta,s)P_{cb}^{MC}(k;s,\eta) && \text{IR (propagator) effects}\\
&+e^\eta\int d^3q\gamma_{acd}(k,q)B_{cdb}^{MC}(q,k;\eta) && \text{Intermediate scales: (resummed) SPT}\\
&-\langle h_a(\mathbf{k},\eta)\varphi_b^{MC}(-\mathbf{k},\eta)\rangle && \text{UV sources (from Nbody)}\\
&+(a\leftrightarrow b)\end{aligned}$$

Improved TRG

Peloso, MP, Viel, Villaescusa-Navarro, in preparation

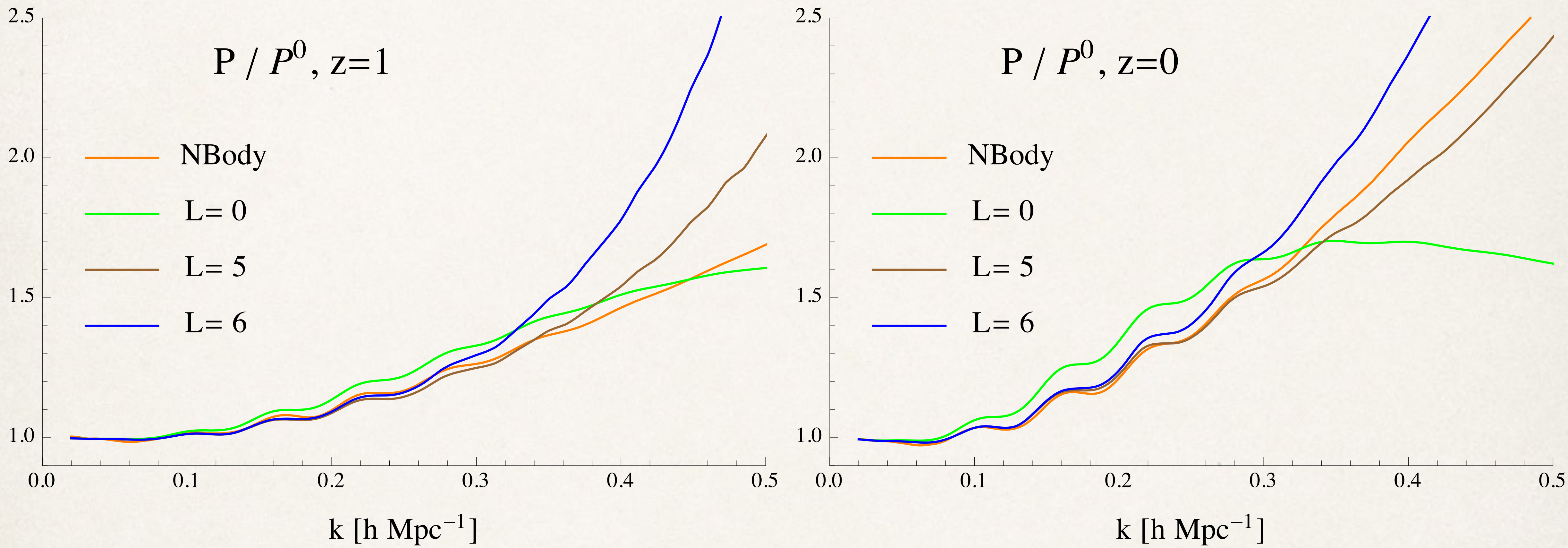
Strategy

IR from Sigma (no parameter)

Intermediate scales from Phi in 1-loop SPT (up to loop momentum L)

UV from N-body sources (for loop momenta larger than L)

The UV impact



L-dependence is a 2-loop effect: renormalisation scale dependence

Should improve at higher orders

L fixed at once for all redshifts

Time consuming as a 1-loop!

Scalar field (axion-like) DM

$$(\Box - m_a^2)\phi = 0 \qquad \Box = -(1-2V)(\partial_t^2 + 3H\partial_t) + a^{-2}(1+2V)\nabla^2 - 4\dot{V}\partial_t$$

$m_a \gg H$ Averaging over fast oscillations: CDM candidate

$$\phi = (m_a\sqrt{2})^{-1}(\psi e^{-im_a t} + \psi^* e^{im_a t})$$

$$i\dot{\psi} - 3iH\psi/2 + (2m_a a^2)^{-1}\nabla^2\psi - m_a V\psi = 0.$$

Shrödinger-Poisson

Perturbations

$$\psi = Re^{iS}$$

$$\rho_a = R^2$$

$$\vec{v_a} = (m_a a)^{-1}\nabla S$$

Madelung

$$\dot{\bar{\rho_a}} + 3H\bar{\rho_a} = 0$$

$$\dot{\delta}_a + a^{-1}\vec{v_a}\cdot\nabla\delta_a + a^{-1}(1+\delta_a)\nabla\cdot\vec{v_a} = 0,$$

$$\dot{\vec{v_a}} + H\vec{v_a} + a^{-1}\left(\vec{v_a}\cdot\nabla\right)\vec{v_a} = -a^{-1}\nabla\left(V+Q\right)$$

$$Q = -\frac{1}{2m_a^2a^2}\frac{\nabla^2\sqrt{1+\delta_a}}{\sqrt{1+\delta_a}}.$$

"Quantum" term, deviations from CDM

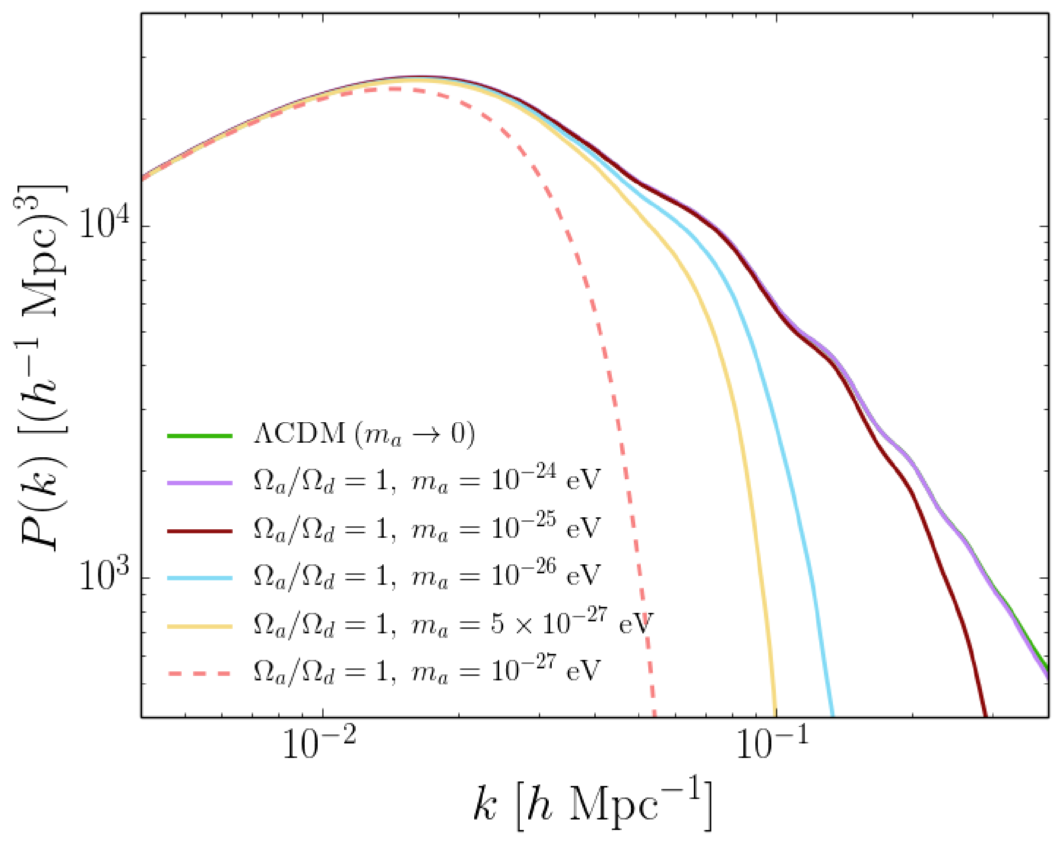
Linear Theory

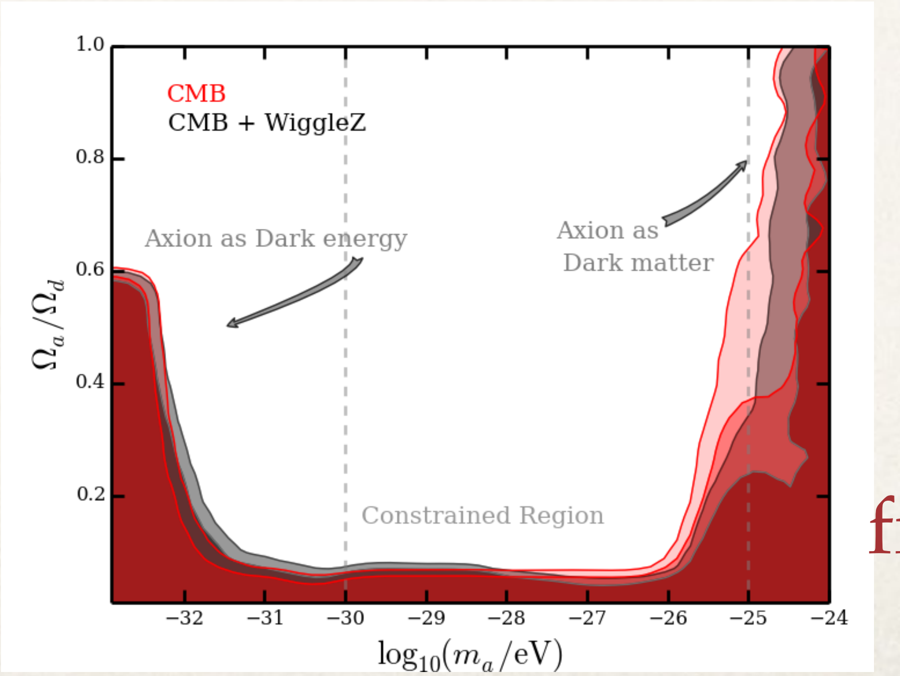
$$\frac{\partial \delta_a(\mathbf{k},\tau)}{\partial \tau} + \theta(\mathbf{k},\tau) = 0$$

$$\frac{\partial \theta(\mathbf{k},\tau)}{\partial \tau} + \mathcal{H}(\tau)\theta(\mathbf{k},\tau) + \frac{3}{2}\mathcal{H}^2(\tau)\delta_a(\mathbf{k},\tau) - \frac{k^4}{4m_a^2a^2}\,\delta_a(\mathbf{k},\tau) = 0$$

$$k_J = \sqrt[4]{6}\sqrt{m_a a \mathcal{H}} \approx 1.6a\sqrt{m_a H}$$

Axion Jeans scale





m > 10^{-24} eV
indistinguishable
from standard CDM

Hlozek, Grin, Marsch, Ferreira 1410.2896

Nonlinear perturbations

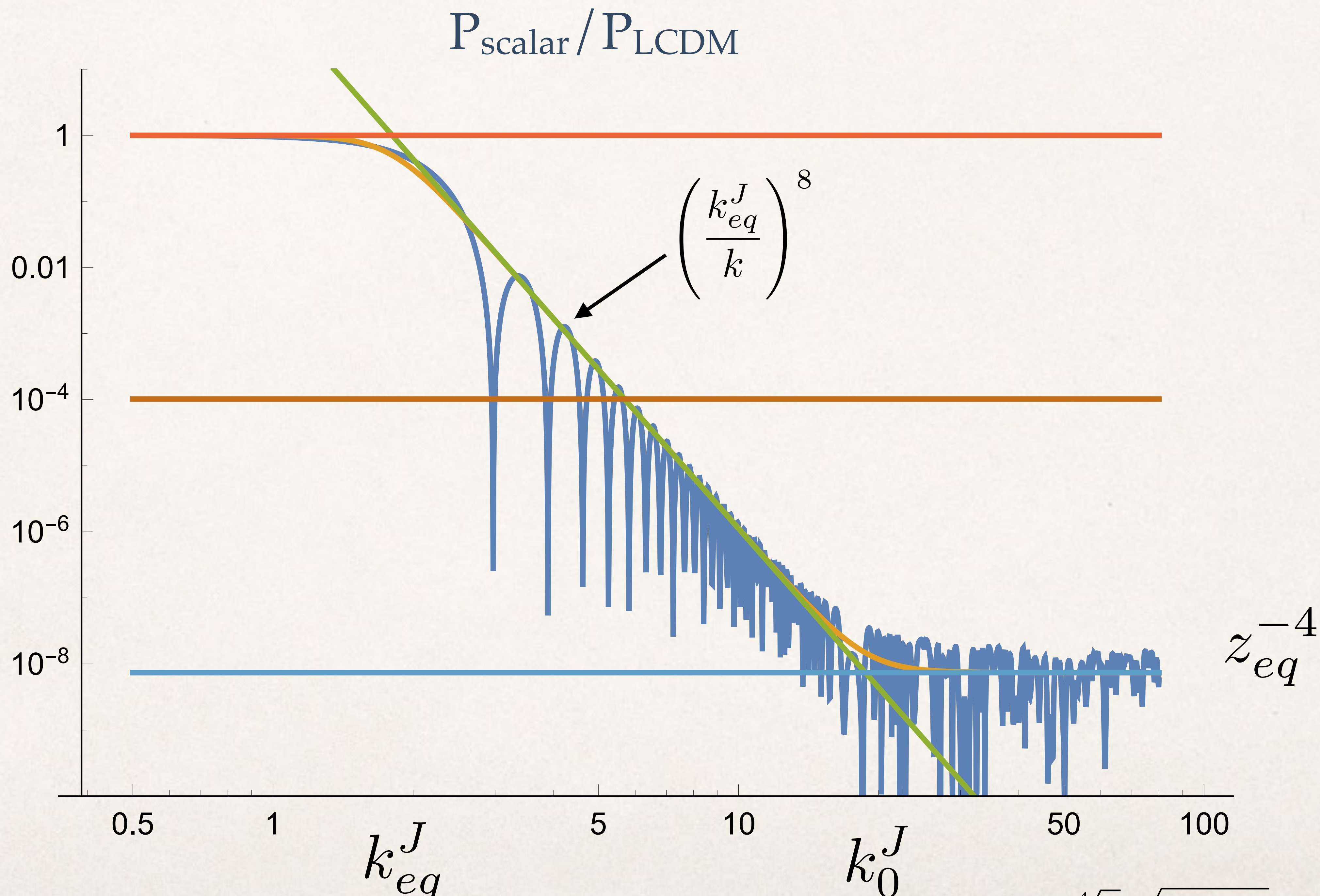
$$\frac{\partial \delta_a(\mathbf{k},\tau)}{\partial \tau} + \theta(\mathbf{k},\tau) + \int d^3\mathbf{p} d^3\mathbf{q} \delta_D(\mathbf{k}-\mathbf{p}-\mathbf{q})\alpha(\mathbf{q},\mathbf{p})\theta(\mathbf{q},\tau)\delta_a(\mathbf{p},\tau)$$

$$\begin{aligned}\frac{\partial \theta(\mathbf{k},\tau)}{\partial \tau} &+ \mathcal{H}(\tau)\theta(\mathbf{k},\tau) + \frac{3}{2}\Omega_m(\tau)\mathcal{H}^2(\tau)\delta_a(\mathbf{k},\tau) - \frac{\mathbf{k}^4}{4m_a^2 a^2}\delta_a(\mathbf{k},\tau) \\ &+ \int d^3\mathbf{p} d^3\mathbf{q} \delta_D(\mathbf{k}-\mathbf{p}-\mathbf{q})\beta(\mathbf{q},\mathbf{p})\theta(\mathbf{p},\tau)\theta(\mathbf{q},\tau) \\ &+ \int d^3\mathbf{p} d^3\mathbf{q} \delta_D(\mathbf{k}-\mathbf{p}-\mathbf{q})\frac{\mathbf{k}^2(\mathbf{k}^2+\mathbf{q}^2+\mathbf{p}^2)}{16 m_a^2 a^2}\delta_a(\mathbf{q},\tau)\delta_a(\mathbf{p},\tau)\end{aligned}$$

From expanding Q to 2nd order
~ k^4: UV catastrophe?

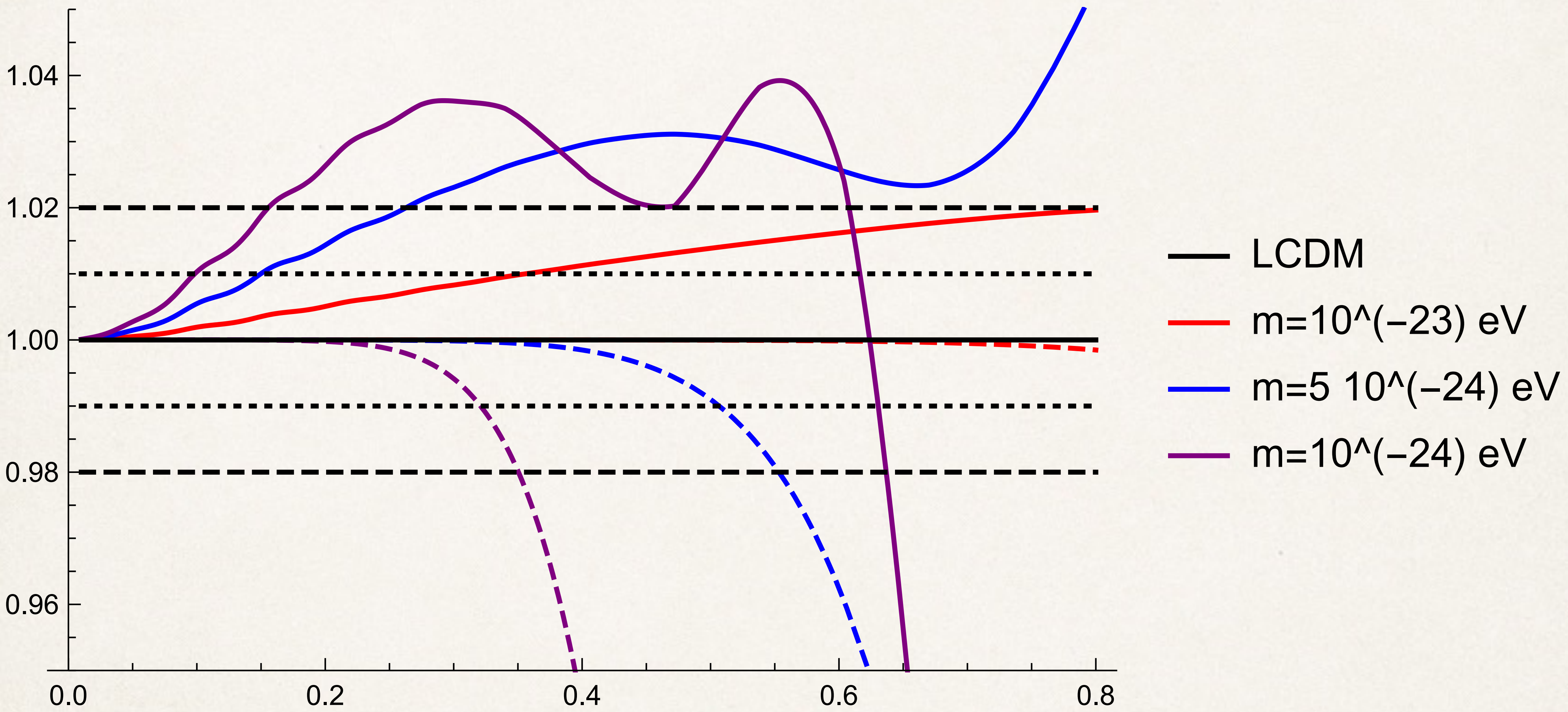
$$\alpha(\mathbf{q},\mathbf{p}) = \frac{(\mathbf{p}+\mathbf{q})\cdot\mathbf{q}}{\mathbf{q}^2} \qquad \beta(\mathbf{q},\mathbf{p}) = \frac{(\mathbf{q}+\mathbf{p})^2\mathbf{q}\cdot\mathbf{p}}{\mathbf{q}^2\mathbf{p}^2}$$

Linear PT cutoff



$$k_J = \sqrt[4]{6}\sqrt{m_a a \mathcal{H}} \approx 1.6a\sqrt{m_a H}$$

TRG results



The UV cutoff acts differently on P13 and P22

m~10^{-23} eV, no effect in linear th., but percent effects by TRG

Summary

- IR is important and is robust
- Intermediate scales treatable by (improved) SPT
- The UV is important but mildly cosmology dependent
- TRG can combine the three, is fast and flexible