# The Lagrangian PT and the integrated PT


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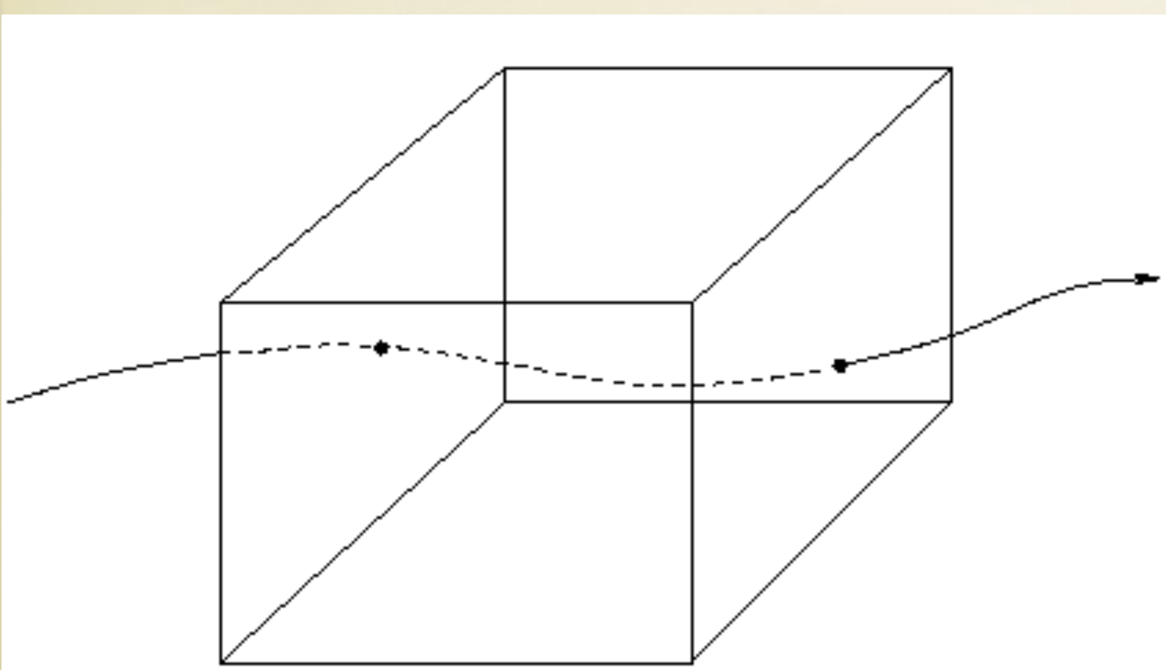
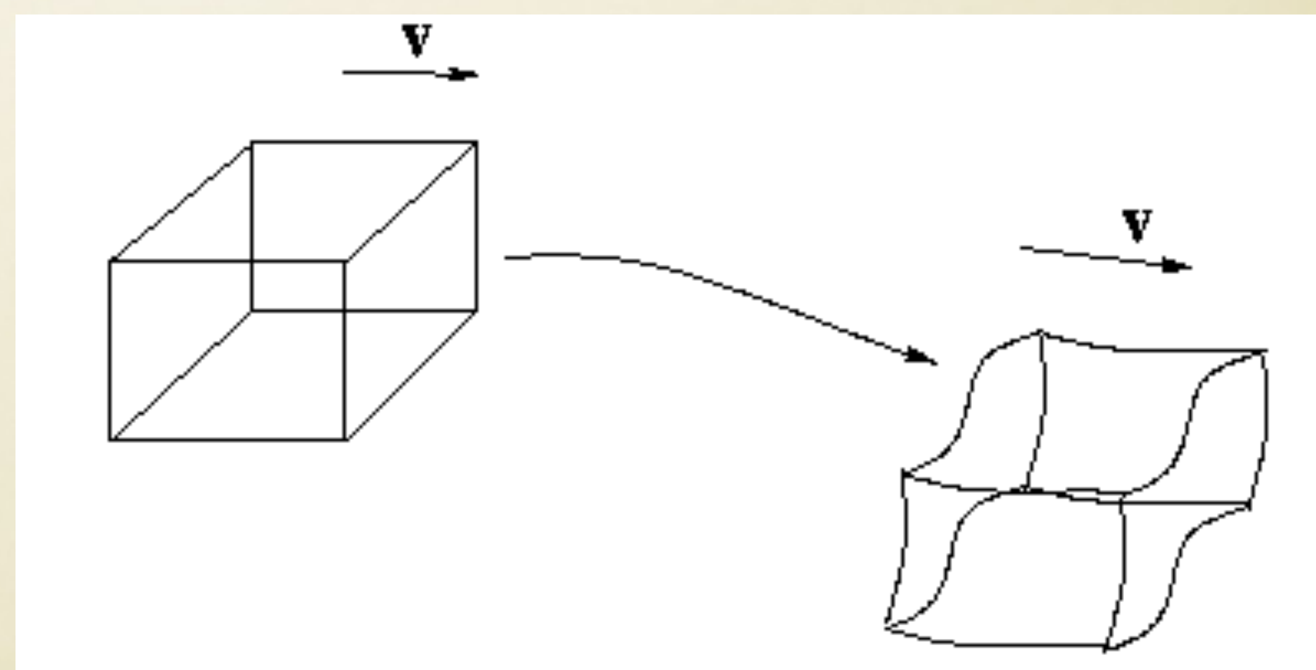
# The Lagrangian Perturbation Theory (LPT)

# Eulerian vs Lagrangian picture

- **Eulerian**
  - density and velocity fields on a fixed space


- **Lagrangian**
  - follows a trajectory of a fluid element

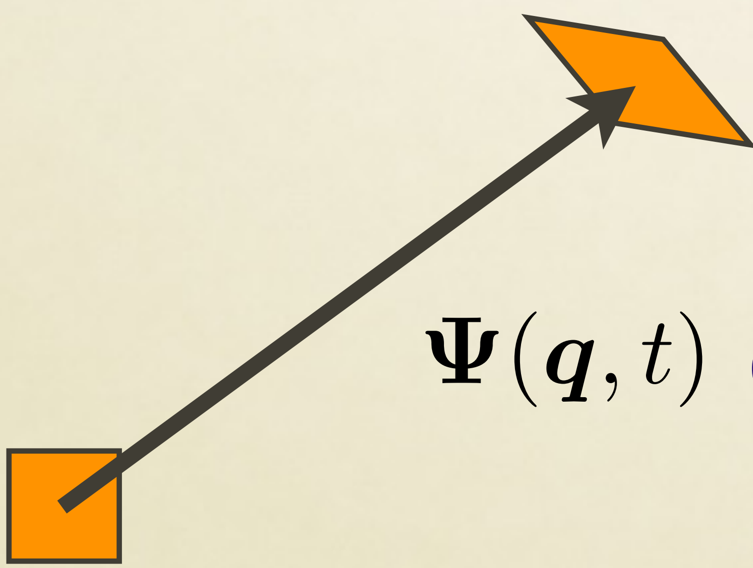





# Variables in the Lagrangian picture

- Fundamental variable in the Lagrangian picture:
  - Displacement of a fluid element from the initial position: Displacement vector field

$$\boldsymbol{\Psi}(\boldsymbol{q}, t) = \boldsymbol{x}(\boldsymbol{q}, t) - \boldsymbol{q}$$



$\boldsymbol{x}(\boldsymbol{q}, t)$ final position

$\boldsymbol{\Psi}(\boldsymbol{q}, t)$ displacement vector

$\boldsymbol{q}$ initial position

# Euleriann vs Lagrangian picture

- **Eulerian**
  - field labels: $\boldsymbol{x}$
  - fundamental variables:

$$\rho(\boldsymbol{x},t) = \bar{\rho}[1+\delta(\boldsymbol{x},t)]$$

$$\boldsymbol{v}(\boldsymbol{x},t)$$

  - gravitational potential

$$\phi(\boldsymbol{x},t) = 4\pi G\bar{\rho}a^2\triangle^{-1}\delta(\boldsymbol{x},t)$$

- **Lagrangian**
  - field labels: $\boldsymbol{q}$
  - fundamental variables:

$$\boldsymbol{\Psi}(\boldsymbol{q},t) = \boldsymbol{x}(\boldsymbol{q},t) - \boldsymbol{q}$$

  - density

$$\rho(\boldsymbol{q},t) = \bar{\rho}\left[\det\left(\frac{\partial\boldsymbol{x}}{\partial\boldsymbol{q}}\right)\right]^{-1}$$

$$= \bar{\rho}\left[\det\left(\boldsymbol{I}+\frac{\partial\boldsymbol{\Psi}}{\partial\boldsymbol{q}}\right)\right]^{-1}$$

  - velocity

$$\boldsymbol{v}(\boldsymbol{q},t) = a\dot{\boldsymbol{x}} = a\dot{\boldsymbol{\Psi}}(\boldsymbol{q},t)$$

# Eulerian vs Lagrangian picture

- **Eulerian**
  - **EoM**

$$\dot{\delta} + \frac{1}{a}\boldsymbol{\nabla}\cdot[(1+\delta)\boldsymbol{v}] = 0$$

$$\dot{\boldsymbol{v}} + \frac{\dot{a}}{a}\boldsymbol{v} + \frac{1}{a}(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = -\frac{1}{a}\boldsymbol{\nabla}\Phi$$

- **Lagrangian**
  - **EoM**

$$\ddot{\boldsymbol{\Psi}} + 2\frac{\dot{a}}{a}\dot{\boldsymbol{\Psi}} = -\frac{1}{a^2}\boldsymbol{\nabla}_{\boldsymbol{x}}\Phi$$

- **Common:**
  - **Poisson equation**

$$\triangle_{\boldsymbol{x}}\Phi = 4\pi G\bar{\rho}a^2\delta(\boldsymbol{x},t)$$

# Zel'dovich approximation

- **Linearize the Lagrangian EoM w.r.t. displacement**

Zel'dovich (1970)


Equations of motion:

$$\ddot{\boldsymbol{\Psi}} + 2\frac{\dot{a}}{a}\dot{\boldsymbol{\Psi}} = -\frac{1}{a^2}\boldsymbol{\nabla}_{\boldsymbol{x}}\Phi$$

$$\triangle_{\boldsymbol{x}}\Phi = 4\pi G\bar{\rho}a^2\delta(\boldsymbol{x},t)$$

Linearization:

$$\delta(\boldsymbol{x},t) = \left[\det\left(\boldsymbol{I} + \frac{\partial\boldsymbol{\Psi}}{\partial\boldsymbol{q}}\right)\right]^{-1} - 1 \simeq -\boldsymbol{\nabla}_{\boldsymbol{q}}\cdot\boldsymbol{\Psi}$$

$$\boldsymbol{\nabla}_{\boldsymbol{x}}\cdot\boldsymbol{\Psi} \simeq \boldsymbol{\nabla}_{\boldsymbol{q}}\cdot\boldsymbol{\Psi}, \quad \boldsymbol{\nabla}_{\boldsymbol{x}}\times\boldsymbol{\Psi} \simeq \boldsymbol{\nabla}_{\boldsymbol{q}}\times\boldsymbol{\Psi}$$

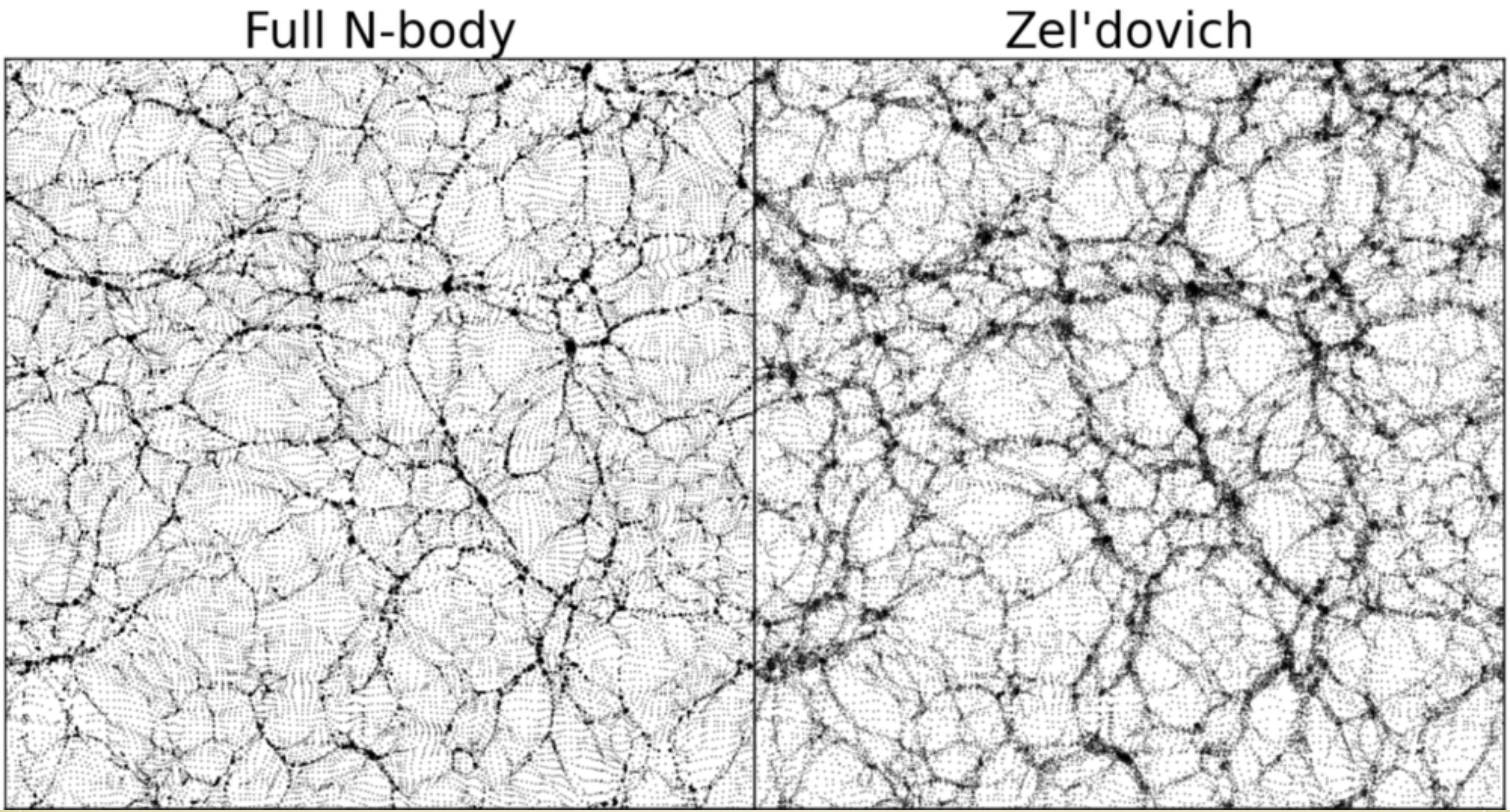
$$\boldsymbol{\nabla}_{\boldsymbol{q}}\cdot\left(\ddot{\boldsymbol{\Psi}} + 2\frac{\dot{a}}{a}\dot{\boldsymbol{\Psi}}\right) = -4\pi G\bar{\rho}\boldsymbol{\nabla}_{\boldsymbol{q}}\cdot\boldsymbol{\Psi}$$

$$\boldsymbol{\nabla}_{\boldsymbol{q}}\times\left(\ddot{\boldsymbol{\Psi}} + 2\frac{\dot{a}}{a}\dot{\boldsymbol{\Psi}}\right) = 0$$

(Taking a growing mode)

$$\boldsymbol{\nabla}_{\boldsymbol{q}}\times\boldsymbol{\Psi} \simeq 0, \quad \boldsymbol{\nabla}_{\boldsymbol{q}}\cdot\boldsymbol{\Psi} \propto D(t)$$

$$\boldsymbol{\Psi} \simeq -D(t)\boldsymbol{\nabla}_{\boldsymbol{q}}\varphi_0(\boldsymbol{q})$$




Neyrinck (2013)

# Lagrangian Perturbation Theory

Buchert (1989); Moutarde+ (1991); Buchert (1992); Buchert & Ehlers (1993); Hivon+ (1995); Catelan (1995); Rampf & Wong (2012); Tatekawa (2013); Zheligovski & Frisch (2014); TM (2015); ...

- **Taking into account the higher-order perturbations in the displacement**

$$\boldsymbol{\Psi} = \sum_{n=1}^{\infty} \boldsymbol{\Psi}^{(n)} = \boldsymbol{\Psi}^{(1)} + \boldsymbol{\Psi}^{(2)} + \boldsymbol{\Psi}^{(3)} + \cdots$$

$$\boldsymbol{\Psi}^{(1)} = -D(t)\boldsymbol{\nabla}\varphi_0(\boldsymbol{q})$$

(First order: Zel'dovich approx.)

$$\boldsymbol{\Psi}^{(2)} = -\frac{1}{2}D_2(t)\boldsymbol{\nabla}\triangle^{-1}\left[\Psi^{(1)}_{i,i}\Psi^{(1)}_{j,j} - \Psi^{(1)}_{i,j}\Psi^{(1)}_{i,j}\right]$$

$$\boldsymbol{\Psi}^{(3)} = -\frac{1}{3!}\Big[D_{3a}(t)\boldsymbol{\nabla}\triangle^{-1}\left(\Psi^{(1)}_{i,i}\Psi^{(2)}_{j,j} - \Psi^{(1)}_{i,j}\Psi^{(2)}_{i,j}\right) + D_{3b}(t)\boldsymbol{\nabla}\triangle^{-1}\det\left(\Psi^{(1)}_{i,j}\right)$$
$$+ D_{3c}(t)\triangle^{-1}\left(\boldsymbol{\Psi}^{(1)}_{,j}\Psi^{(2)}_{i,j} - \Psi^{(1)}_{i,j}\boldsymbol{\Psi}^{(2)}_{,j}\right)_{,i}\Big]$$

$$\vdots$$

$$\left(D_2 \simeq \frac{3}{7}D^2, D_{3a} \simeq -\frac{10}{7}D^2, D_{3b} \simeq 2D^2, D_{3c} \simeq -\frac{6}{7}D^2, \ldots\right)$$

# Recursive solutions (general)

TM (2015)

$$\ddot{\boldsymbol{x}} + 2H\dot{\boldsymbol{x}} = -\frac{1}{a^2}\boldsymbol{\nabla}_x\phi(\boldsymbol{x},t), \qquad \triangle_x\phi(\boldsymbol{x},t) = 4\pi G\bar{\rho}a^2\delta(\boldsymbol{x},t),$$

$$\hat{\mathcal{T}} \equiv \frac{\partial^2}{\partial t^2} + 2H\frac{\partial}{\partial t}, \qquad \boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \boldsymbol{\Psi}(\boldsymbol{q},t).$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{\Psi} = D_+(t)A_+ + D_-(t)A_- - \left(\hat{\mathcal{T}} - 4\pi G\bar{\rho}\right)^{-1}\left[\varepsilon_{ijk}\varepsilon_{ipq}\Psi_{j,p}\left(\hat{\mathcal{T}} - 2\pi G\bar{\rho}\right)\Psi_{k,q} + \frac{1}{2}\varepsilon_{ijk}\varepsilon_{pqr}\Psi_{i,p}\Psi_{j,q}\left(\hat{\mathcal{T}} - \frac{4\pi G}{3}\bar{\rho}\right)\Psi_{k,r}\right],$$

$$\boldsymbol{\nabla}\times\boldsymbol{\Psi} = \boldsymbol{B}_0 + E_-(t)\boldsymbol{B}_- + \hat{\mathcal{T}}^{-1}\left(\boldsymbol{\nabla}\Psi_i \times \hat{\mathcal{T}}\boldsymbol{\nabla}\Psi_i\right),$$

$$\boldsymbol{\Psi} = \sum_{n=1}^{\infty}\boldsymbol{\Psi}^{(n)} = \boldsymbol{\Psi}^{(1)} + \boldsymbol{\Psi}^{(2)} + \boldsymbol{\Psi}^{(3)} + \cdots, \qquad \boldsymbol{\Psi} = \Delta^{-1}\left[\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{\Psi}) - \boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{\Psi})\right],$$

# Recursive solutions (growing modes)

$$\tilde{\boldsymbol{\Psi}}^{(n)}(\boldsymbol{k},t) = \frac{iD^n}{n!}\int_{\boldsymbol{k}_{1\cdots n}=\boldsymbol{k}} \boldsymbol{L}_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)\delta_0(\boldsymbol{k}_1)\cdots\delta_0(\boldsymbol{k}_n).$$

$$\boldsymbol{k}_{1\cdots n} \equiv \boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n$$

$$\boldsymbol{L}_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = \frac{1}{k_{1\cdots n}{}^2}\left[\boldsymbol{k}_{1\cdots n} S_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) + \boldsymbol{k}_{1\cdots n} \times \boldsymbol{T}_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)\right].$$

$$U(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{|\boldsymbol{k}_1\times\boldsymbol{k}_2|^2}{k_1{}^2k_2{}^2} = 1 - \left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1k_2}\right)^2,$$

$$V(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{|\boldsymbol{k}_1\cdot(\boldsymbol{k}_2\times\boldsymbol{k}_3)|^2}{k_1{}^2k_2{}^2k_2{}^3} = 1 - \left(\frac{\boldsymbol{k}_1\cdot\boldsymbol{k}_2}{k_1k_2}\right)^2 - \left(\frac{\boldsymbol{k}_2\cdot\boldsymbol{k}_3}{k_2k_3}\right)^2 - \left(\frac{\boldsymbol{k}_3\cdot\boldsymbol{k}_1}{k_3k_1}\right)^2 + 2\frac{(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)(\boldsymbol{k}_2\cdot\boldsymbol{k}_3)(\boldsymbol{k}_3\cdot\boldsymbol{k}_1)}{k_1{}^2k_2{}^2k_3{}^2},$$

$$\boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{(\boldsymbol{k}_1\times\boldsymbol{k}_2)(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)}{k_1{}^2k_2{}^2}.$$

$$S_1(\boldsymbol{k}) = 1, \quad \boldsymbol{T}_1(\boldsymbol{k}) = \boldsymbol{0}.$$

$$S_2(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{3}{7}U(\boldsymbol{k}_1,\boldsymbol{k}_2), \quad \boldsymbol{T}_2(\boldsymbol{k}_1,\boldsymbol{k}_2) = \boldsymbol{0}.$$

$$S_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{5}{3}U(\boldsymbol{k}_1,\boldsymbol{k}_{23})S_2(\boldsymbol{k}_2,\boldsymbol{k}_3) - \frac{1}{3}V(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3),$$

$$\boldsymbol{T}_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_{23})S_2(\boldsymbol{k}_2,\boldsymbol{k}_3).$$

$$S_4(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_4) = \frac{28}{11}\Big[U(\boldsymbol{k}_1,\boldsymbol{k}_{234})S_3(\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) - \boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_{234})\cdot\boldsymbol{T}_3(\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)\Big] + \frac{17}{11}U(\boldsymbol{k}_{12},\boldsymbol{k}_{34})S_2(\boldsymbol{k}_1,\boldsymbol{k}_2)S_2(\boldsymbol{k}_3,\boldsymbol{k}_4) - \frac{26}{11}V(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_{34})S_2(\boldsymbol{k}_3,\boldsymbol{k}_4),$$

$$\boldsymbol{T}_4(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_4) = 2\Big[\boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_{234})S_3(\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) + \frac{\boldsymbol{k}_1\times\boldsymbol{k}_{234}}{k_1{}^2k_{234}{}^2}(\boldsymbol{k}_1\times\boldsymbol{k}_{234})\cdot\boldsymbol{T}_3(\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)\Big].$$

$$S_5(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_5) = \frac{45}{13}\Big[U(\boldsymbol{k}_1,\boldsymbol{k}_{2345})S_4(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_5) - \boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_{2345})\cdot\boldsymbol{T}_4(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_5)\Big] + \frac{70}{13}S_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\Big[U(\boldsymbol{k}_{12},\boldsymbol{k}_{345})S_3(\boldsymbol{k}_3,\boldsymbol{k}_4,\boldsymbol{k}_5) - \boldsymbol{W}(\boldsymbol{k}_{12},\boldsymbol{k}_{345})\cdot\boldsymbol{T}_3(\boldsymbol{k}_3,\boldsymbol{k}_4,\boldsymbol{k}_5)\Big] - \frac{60}{13}\left\{V(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_{345})S_3(\boldsymbol{k}_3,\boldsymbol{k}_4,\boldsymbol{k}_5) + \frac{(\boldsymbol{k}_1\times\boldsymbol{k}_2)\cdot\boldsymbol{k}_{345}}{k_1{}^2k_2{}^2k_{345}{}^2}\left[(\boldsymbol{k}_1\times\boldsymbol{k}_2)\times\boldsymbol{k}_{345}\right]\cdot\boldsymbol{T}_3(\boldsymbol{k}_3,\boldsymbol{k}_4,\boldsymbol{k}_5)\right\} - \frac{75}{13}V(\boldsymbol{k}_1,\boldsymbol{k}_{23},\boldsymbol{k}_{45})S_2(\boldsymbol{k}_2,\boldsymbol{k}_3)S_2(\boldsymbol{k}_4,\boldsymbol{k}_5),$$

$$\boldsymbol{T}_5(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_5) = 3\Big[\boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_{2345})S_4(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_5) + \frac{\boldsymbol{k}_1\times\boldsymbol{k}_{2345}}{k_1{}^2k_{2345}{}^2}(\boldsymbol{k}_1\times\boldsymbol{k}_{2345})\cdot\boldsymbol{T}_4(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_5)\Big] + 2S_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\Big[\boldsymbol{W}(\boldsymbol{k}_{12},\boldsymbol{k}_{345})S_3(\boldsymbol{k}_3,\boldsymbol{k}_4,\boldsymbol{k}_5) + \frac{\boldsymbol{k}_{12}\times\boldsymbol{k}_{345}}{k_{12}{}^2k_{345}{}^2}(\boldsymbol{k}_{12}\times\boldsymbol{k}_{345})\cdot\boldsymbol{T}_3(\boldsymbol{k}_3,\boldsymbol{k}_4,\boldsymbol{k}_5)\Big].$$

$$
\begin{aligned}
S_6(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_6) = \frac{22}{5}\Big[& U(\boldsymbol{k}_1,\boldsymbol{k}_{23456})S_5(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_6) - \boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_{23456})\cdot\boldsymbol{T}_5(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_6)\Big] \\
&+ \frac{43}{5}S_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\Big[U(\boldsymbol{k}_{12},\boldsymbol{k}_{3456})S_4(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_6) - \boldsymbol{W}(\boldsymbol{k}_{12},\boldsymbol{k}_{3456})\cdot\boldsymbol{T}_4(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_6)\Big] \\
&+ \frac{26}{5}\Big[S_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\left[U(\boldsymbol{k}_{123},\boldsymbol{k}_{456})S_3(\boldsymbol{k}_4,\boldsymbol{k}_5,\boldsymbol{k}_6) - 2\boldsymbol{W}(\boldsymbol{k}_{123},\boldsymbol{k}_{456})\cdot\boldsymbol{T}_3(\boldsymbol{k}_4,\boldsymbol{k}_5,\boldsymbol{k}_6)\right] \\
&\qquad + \frac{\boldsymbol{k}_{123}\times\boldsymbol{k}_{456}}{{k_{123}}^2{k_{456}}^2}\cdot\left\{[\boldsymbol{k}_{123}\times\boldsymbol{T}_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)]\times[\boldsymbol{k}_{456}\times\boldsymbol{T}_3(\boldsymbol{k}_4,\boldsymbol{k}_5,\boldsymbol{k}_6)]\right\}\Big] \\
&- \frac{39}{5}\left\{V(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_{3456})S_4(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_6) + \frac{(\boldsymbol{k}_1\times\boldsymbol{k}_2)\cdot\boldsymbol{k}_{3456}}{{k_1}^2{k_2}^2{k_{3456}}^2}\left[(\boldsymbol{k}_1\times\boldsymbol{k}_2)\times\boldsymbol{k}_{3456}\right]\cdot\boldsymbol{T}_4(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_6)\right\} \\
&- \frac{124}{5}S_2(\boldsymbol{k}_2,\boldsymbol{k}_3)\left\{V(\boldsymbol{k}_1,\boldsymbol{k}_{23},\boldsymbol{k}_{456})S_3(\boldsymbol{k}_4,\boldsymbol{k}_5,\boldsymbol{k}_6) + \frac{(\boldsymbol{k}_1\times\boldsymbol{k}_{23})\cdot\boldsymbol{k}_{456}}{{k_1}^2{k_{23}}^2{k_{456}}^2}\left[(\boldsymbol{k}_1\times\boldsymbol{k}_{23})\times\boldsymbol{k}_{456}\right]\cdot\boldsymbol{T}_3(\boldsymbol{k}_4,\boldsymbol{k}_5,\boldsymbol{k}_6)\right\} \\
&- \frac{27}{5}V(\boldsymbol{k}_{12},\boldsymbol{k}_{34},\boldsymbol{k}_{56})S_2(\boldsymbol{k}_1,\boldsymbol{k}_2)S_2(\boldsymbol{k}_3,\boldsymbol{k}_4)S_2(\boldsymbol{k}_5,\boldsymbol{k}_6), \qquad \text{(B1}
\end{aligned}
$$

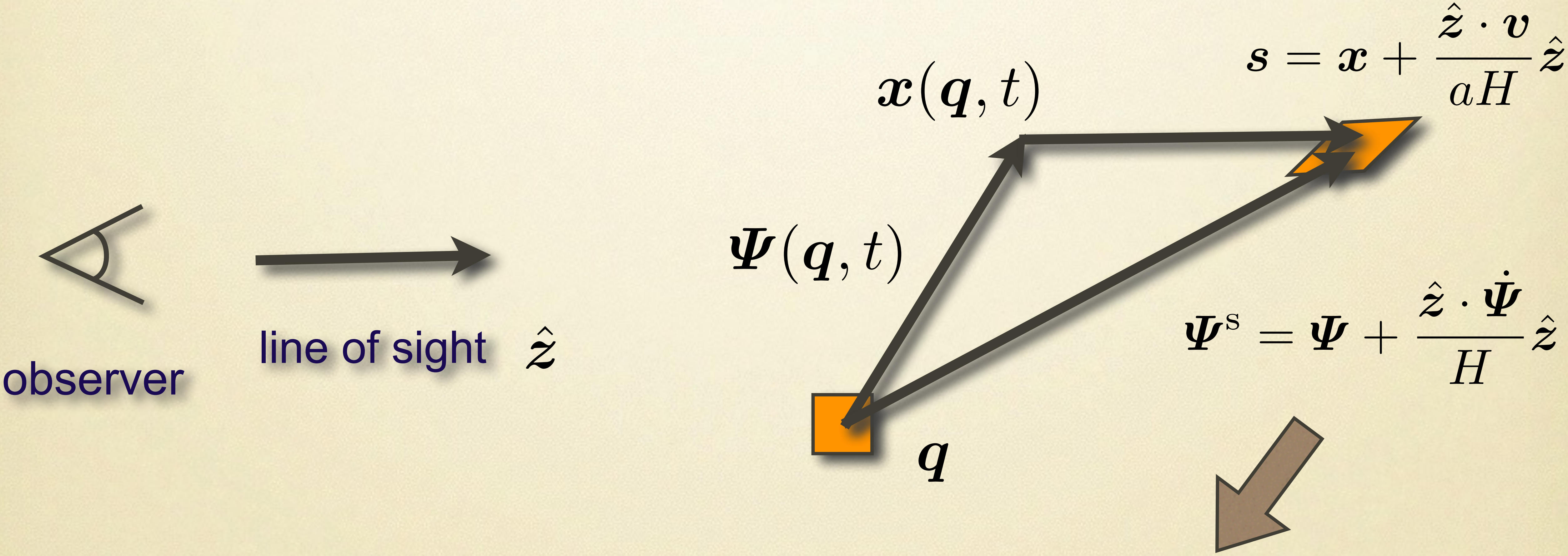
$$
\begin{aligned}
\boldsymbol{T}_6(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_6) = 4\Big[&\boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_{23456})S_5(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_6) + \frac{\boldsymbol{k}_1\times\boldsymbol{k}_{23456}}{{k_1}^2{k_{23456}}^2}(\boldsymbol{k}_1\times\boldsymbol{k}_{23456})\cdot\boldsymbol{T}_5(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_6)\Big] \\
&+ 5S_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\Big[\boldsymbol{W}(\boldsymbol{k}_{12},\boldsymbol{k}_{3456})S_4(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_6) + \frac{\boldsymbol{k}_{12}\times\boldsymbol{k}_{3456}}{{k_{12}}^2{k_{3456}}^2}(\boldsymbol{k}_{12}\times\boldsymbol{k}_{3456})\cdot\boldsymbol{T}_4(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_6)\Big]. \qquad \text{(B1}
\end{aligned}
$$

$$
\begin{aligned}
S_7(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_7) = \frac{91}{17}\Big[&U(\boldsymbol{k}_1,\boldsymbol{k}_{234567})S_6(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_7) - \boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_{234567})\cdot\boldsymbol{T}_6(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_7)\Big]\\
&+\frac{217}{17}S_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\Big[U(\boldsymbol{k}_{12},\boldsymbol{k}_{34567})S_5(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_7) - \boldsymbol{W}(\boldsymbol{k}_{12},\boldsymbol{k}_{34567})\cdot\boldsymbol{T}_5(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_7)\Big]\\
&+\frac{315}{17}\Big[U(\boldsymbol{k}_{123},\boldsymbol{k}_{4567})S_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)S_4(\boldsymbol{k}_4,\ldots,\boldsymbol{k}_7)\\
&\qquad-\boldsymbol{W}(\boldsymbol{k}_{123},\boldsymbol{k}_{4567})\cdot[S_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\boldsymbol{T}_4(\boldsymbol{k}_4,\ldots,\boldsymbol{k}_7) - \boldsymbol{T}_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)S_4(\boldsymbol{k}_5,\ldots,\boldsymbol{k}_7)]\\
&\qquad+\frac{\boldsymbol{k}_{123}\times\boldsymbol{k}_{4567}}{k_{123}{}^2k_{4567}{}^2}\cdot\Big\{[\boldsymbol{k}_{123}\times\boldsymbol{T}_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)]\times[\boldsymbol{k}_{4567}\times\boldsymbol{T}_4(\boldsymbol{k}_4,\ldots,\boldsymbol{k}_7)]\Big\}\Big]\\
&-\frac{203}{17}\Big\{V(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_{34567})S_4(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_7) + \frac{(\boldsymbol{k}_1\times\boldsymbol{k}_2)\cdot\boldsymbol{k}_{34567}}{k_1{}^2k_2{}^2k_{34567}{}^2}[(\boldsymbol{k}_1\times\boldsymbol{k}_2)\times\boldsymbol{k}_{34567}]\cdot\boldsymbol{T}_5(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_7)\Big\}\\
&-\frac{805}{17}S_2(\boldsymbol{k}_2,\boldsymbol{k}_3)\Big\{V(\boldsymbol{k}_1,\boldsymbol{k}_{23},\boldsymbol{k}_{4567})S_4(\boldsymbol{k}_4,\ldots,\boldsymbol{k}_7) + \frac{(\boldsymbol{k}_1\times\boldsymbol{k}_{23})\cdot\boldsymbol{k}_{4567}}{k_1{}^2k_{23}{}^2k_{4567}{}^2}[(\boldsymbol{k}_1\times\boldsymbol{k}_{23})\times\boldsymbol{k}_{4567}]\cdot\boldsymbol{T}_4(\boldsymbol{k}_4,\ldots,\boldsymbol{k}_7)]\Big\}\\
&-\frac{490}{17}\Big[V(\boldsymbol{k}_1,\boldsymbol{k}_{234},\boldsymbol{k}_{567})S_3(\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)S_3(\boldsymbol{k}_5,\boldsymbol{k}_6,\boldsymbol{k}_7)\\
&\qquad+2S_3(\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)\frac{(\boldsymbol{k}_1\times\boldsymbol{k}_{234})\cdot\boldsymbol{k}_{567}}{k_1{}^2k_{234}{}^2k_{567}{}^2}[(\boldsymbol{k}_1\times\boldsymbol{k}_{234})\times\boldsymbol{k}_{567}]\cdot\boldsymbol{T}_3(\boldsymbol{k}_5,\boldsymbol{k}_6,\boldsymbol{k}_7)]\\
&\qquad+\frac{(\boldsymbol{k}_1\times\boldsymbol{k}_{234})\cdot\boldsymbol{k}_{567}}{k_1{}^2k_{234}{}^2k_{567}{}^2}\boldsymbol{k}_1\cdot\Big\{[\boldsymbol{k}_{234}\times\boldsymbol{T}_3(\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)]\times[\boldsymbol{k}_{567}\times\boldsymbol{T}_3(\boldsymbol{k}_5,\boldsymbol{k}_6,\boldsymbol{k}_7)]\Big\}\Big]\\
&-\frac{665}{17}S_2(\boldsymbol{k}_1,\boldsymbol{k}_2)S_2(\boldsymbol{k}_3,\boldsymbol{k}_4)\Big\{V(\boldsymbol{k}_{12},\boldsymbol{k}_{34},\boldsymbol{k}_{567})S_3(\boldsymbol{k}_5,\boldsymbol{k}_6,\boldsymbol{k}_7)\\
&\qquad\qquad+\frac{(\boldsymbol{k}_{12}\times\boldsymbol{k}_{34})\cdot\boldsymbol{k}_{567}}{k_{12}{}^2k_{34}{}^2k_{567}{}^2}[(\boldsymbol{k}_{12}\times\boldsymbol{k}_{34})\times\boldsymbol{k}_{567}]\cdot\boldsymbol{T}_3(\boldsymbol{k}_5,\boldsymbol{k}_6,\boldsymbol{k}_7)]\Big\},
\end{aligned}
\tag{B17}
$$

$$
\begin{aligned}
\boldsymbol{T}_7(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_7) = 5\Big[&\boldsymbol{W}(\boldsymbol{k}_1,\boldsymbol{k}_{234567})S_6(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_7) + \frac{\boldsymbol{k}_1\times\boldsymbol{k}_{234567}}{k_1{}^2k_{234567}{}^2}(\boldsymbol{k}_1\times\boldsymbol{k}_{234567})\cdot\boldsymbol{T}_6(\boldsymbol{k}_2,\ldots,\boldsymbol{k}_7)\Big]\\
&+9S_2(\boldsymbol{k}_1,\boldsymbol{k}_2)\Big[\boldsymbol{W}(\boldsymbol{k}_{12},\boldsymbol{k}_{34567})S_5(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_7) + \frac{\boldsymbol{k}_{12}\times\boldsymbol{k}_{34567}}{k_{12}{}^2k_{34567}{}^2}(\boldsymbol{k}_{12}\times\boldsymbol{k}_{34567})\cdot\boldsymbol{T}_5(\boldsymbol{k}_3,\ldots,\boldsymbol{k}_7)\Big].\\
&+5\Big[\boldsymbol{W}(\boldsymbol{k}_{123},\boldsymbol{k}_{4567})S_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)S_4(\boldsymbol{k}_4,\ldots,\boldsymbol{k}_7)\\
&\qquad+\frac{\boldsymbol{k}_{123}\times\boldsymbol{k}_{4567}}{k_{123}{}^2k_{4567}{}^2}\Big\{(\boldsymbol{k}_{123}\times\boldsymbol{k}_{4567})\cdot[S_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\boldsymbol{T}_4(\boldsymbol{k}_4,\ldots,\boldsymbol{k}_7) - \boldsymbol{T}_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)S_4(\boldsymbol{k}_4,\ldots,\boldsymbol{k}_7)]\\
&\qquad\qquad+[\boldsymbol{k}_{123}\times\boldsymbol{T}_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)]\cdot[\boldsymbol{k}_{4567}\times\boldsymbol{T}_4(\boldsymbol{k}_4,\ldots,\boldsymbol{k}_7)]\Big\}\Big].
\end{aligned}
\tag{B18}
$$

# Redshift-space distortions in Lagrangian PT

- **Lagrangian variables and redshift space distortions**



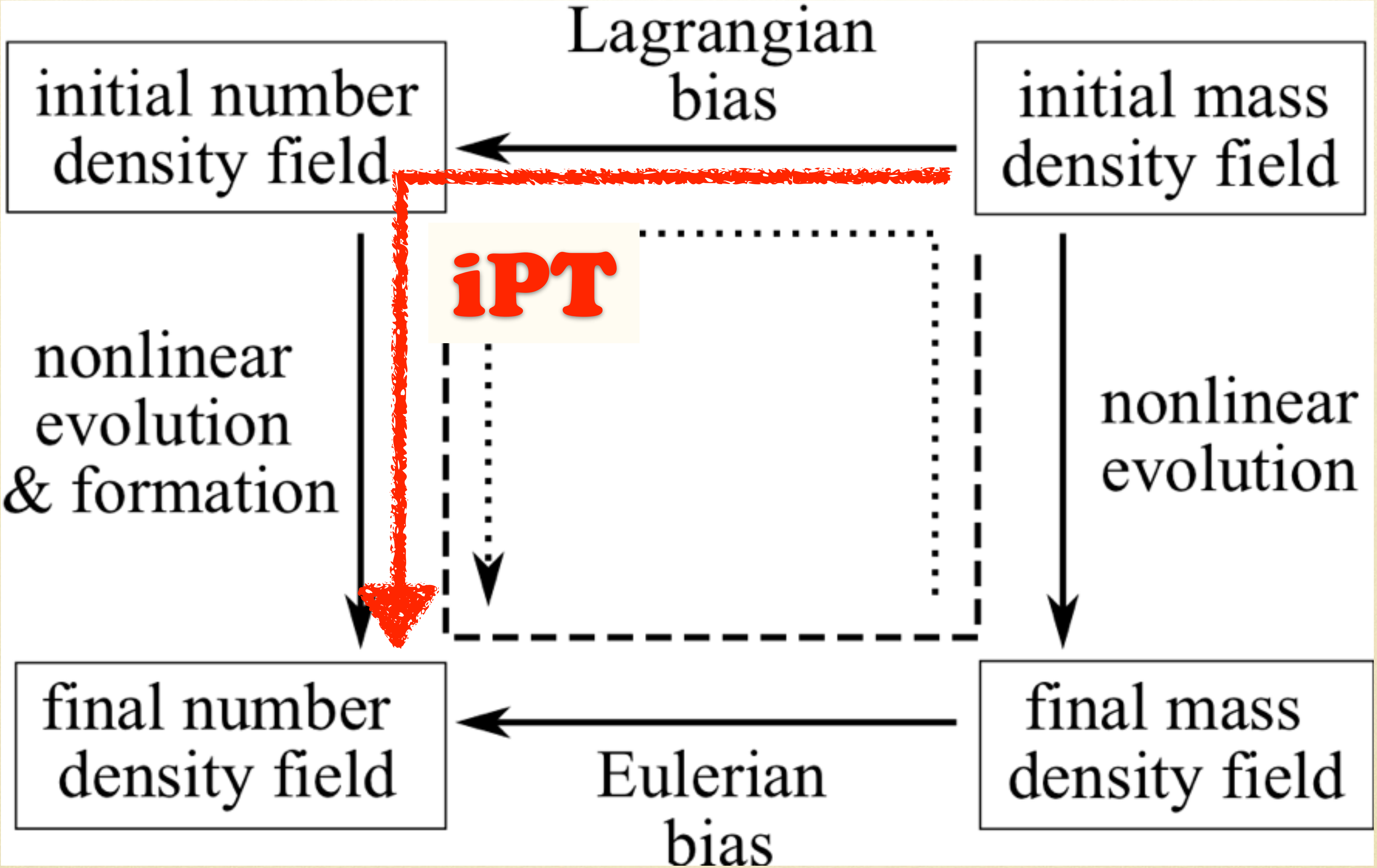

$$\boldsymbol{\Psi}^{\mathrm{s}(n)} = \boldsymbol{\Psi}^{(n)} + \frac{\hat{\boldsymbol{z}}\cdot\dot{\boldsymbol{\Psi}}^{(n)}}{H}\hat{\boldsymbol{z}} \simeq \boldsymbol{\Psi}^{(n)} + nf(\hat{\boldsymbol{z}}\cdot\boldsymbol{\Psi}^{(n)})\hat{\boldsymbol{z}}$$

# The integrated Perturbation Theory (iPT)

# iPT

- integrated Perturbation Theory
  - integration of:
    - nonlinear perturbation theory
    - nonlinear redshift-space distortion
    - nonlinear bias (nonlocal in general)
    - primordial non-Gaussianity
  - iPT does not primarily intend to extrapolate PT into strongly nonlinear regime
  - It provides a basic framework to predict observable quantities from the LPT

# Dynamical evolution of Lagrangian bias



# iPT in a nutshell

- **The relation between Eulerian density fluctuations and Lagrangian variables**

$$1+\delta_{\rm X}(\boldsymbol{x}) = \int d^3q \left[1+\delta_{\rm X}^{\rm L}(\boldsymbol{q})\right]\delta_{\rm D}^3[\boldsymbol{x}-\boldsymbol{q}-\boldsymbol{\Psi}(\boldsymbol{q})]$$

Eulerian density field

Biased field in Lagrangian space

displacement (& redshift distortions)

- **Perturbative expansion in Fourier space**

Kernel of the Lagrangian bias

$$\delta_{\rm X}^{\rm L}(\boldsymbol{k}) = \sum_{n=1}^{\infty}\frac{1}{n!}\int\frac{d^3k_1}{(2\pi)^3}\cdots\frac{d^3k_n}{(2\pi)^3}(2\pi)^3\delta_{\rm D}^3(\boldsymbol{k}_{1\cdots n}-\boldsymbol{k})b_n^{\rm L}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)\delta_{\rm L}(\boldsymbol{k}_1)\cdots\delta_{\rm L}(\boldsymbol{k}_n)$$

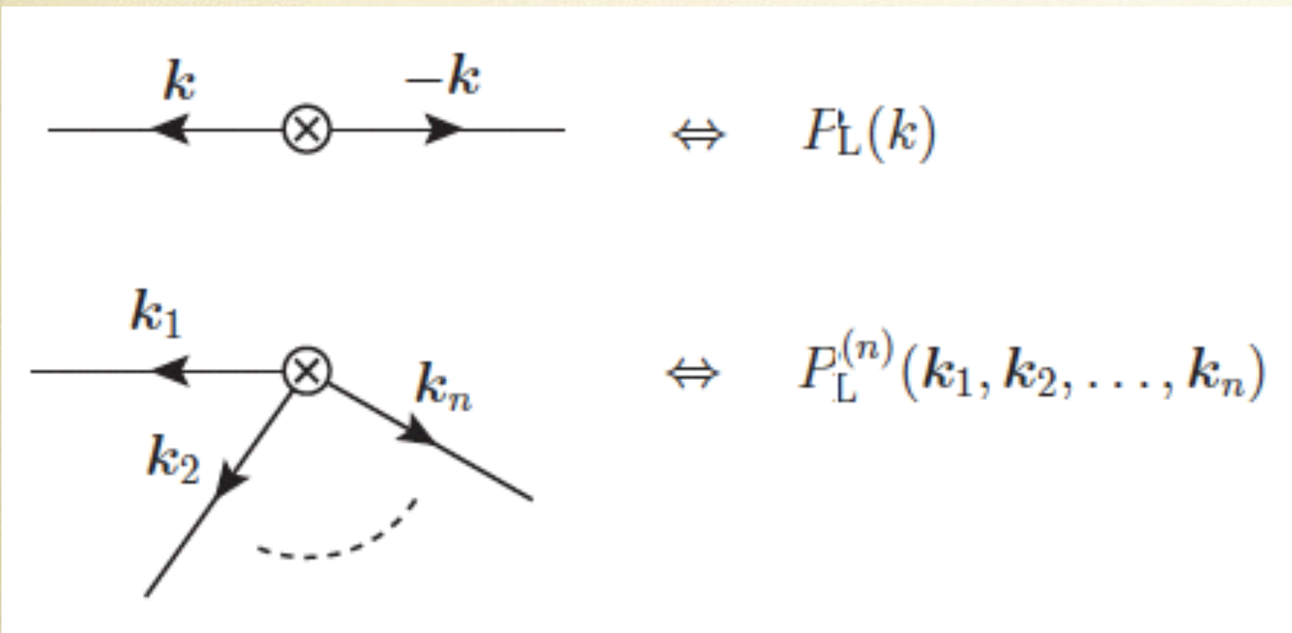
$$\tilde{\boldsymbol{\Psi}}(\boldsymbol{k}) = \sum_{n=1}^{\infty}\frac{i}{n!}\int\frac{d^3k_1}{(2\pi)^3}\cdots\frac{d^3k_n}{(2\pi)^3}(2\pi)^3\delta_{\rm D}^3(\boldsymbol{k}_{1\cdots n}-\boldsymbol{k})\boldsymbol{L}_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)\delta_{\rm L}(\boldsymbol{k}_1)\cdots\delta_{\rm L}(\boldsymbol{k}_n)$$

Kernel of the displacement field (& redshift distortions)

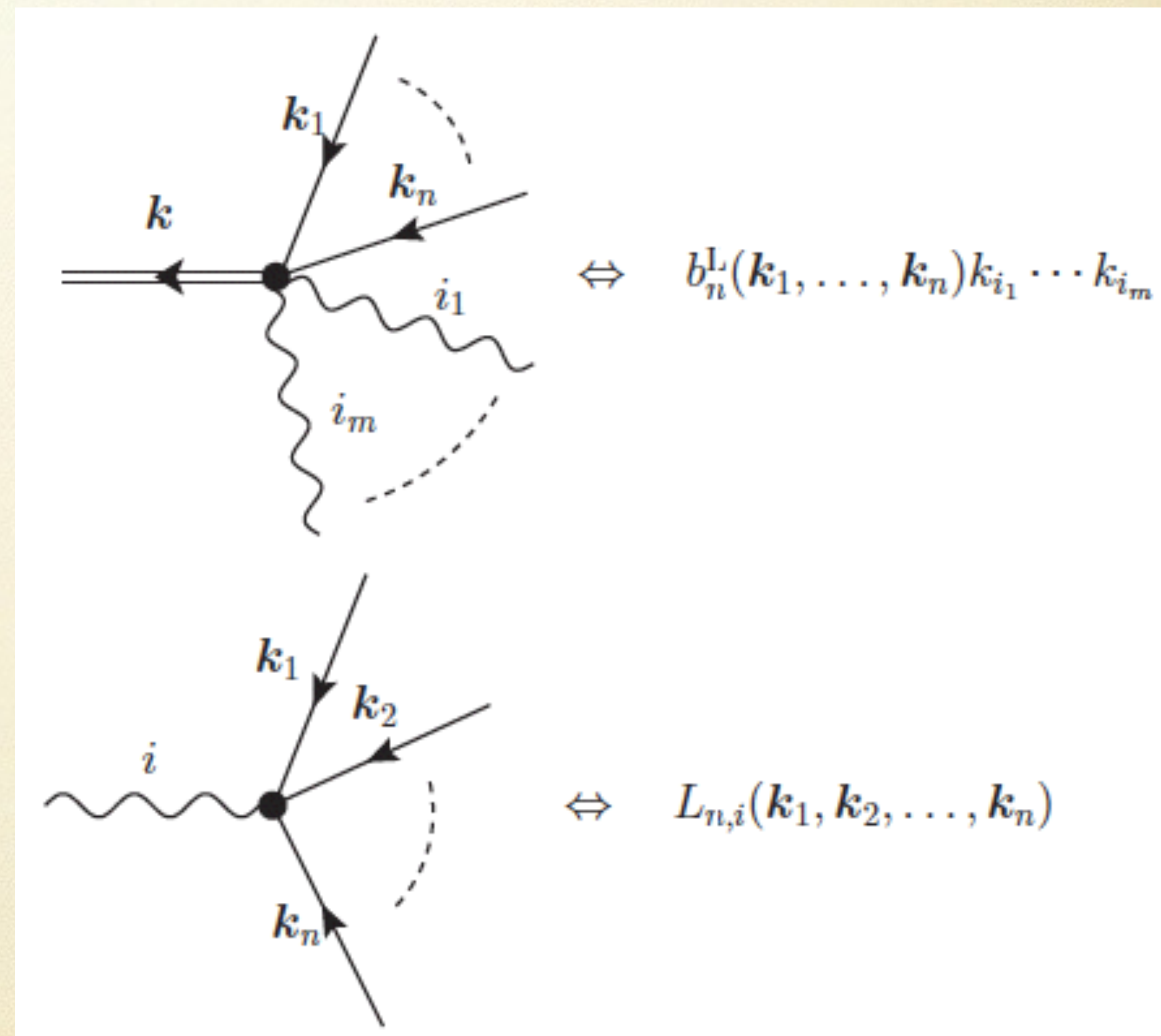
$$\boldsymbol{k}_{1\cdots n} \equiv \boldsymbol{k}_1+\cdots+\boldsymbol{k}_n$$

# Diagrams in iPT

**Primordial spectra**




**Vertices in Lagrangian PT**



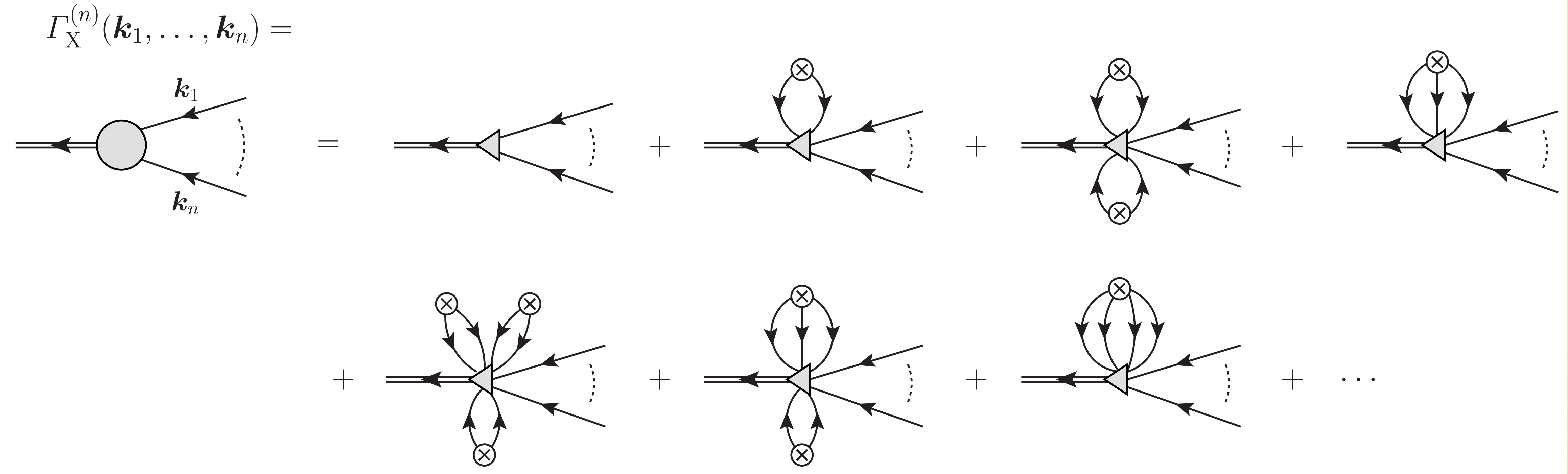

**can naturally deal with RSD and nG**

# Multi-point propagator

TM (1995); Crocce & Scoccimarro (2006); Bernardeau et al. (2008); TM (2011)

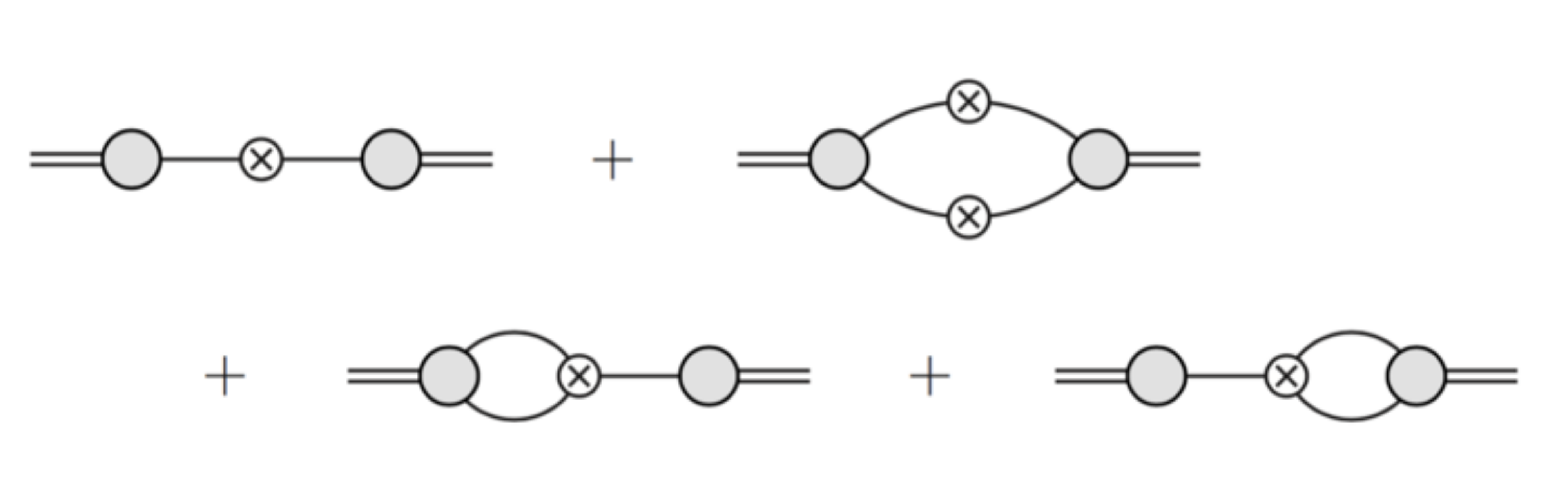
- Density sector of multi-point propagator with nonlocal bias and RSD

$$\left\langle \frac{\delta^n \delta_X(\boldsymbol{k})}{\delta\delta_L(\boldsymbol{k}_1)\cdots\delta\delta_L(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta_D^3(\boldsymbol{k}-\boldsymbol{k}_{1\cdots n})\Gamma_X^{(n)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)$$



# One-loop power spectrum

- In the formalism of multi-point propagator,

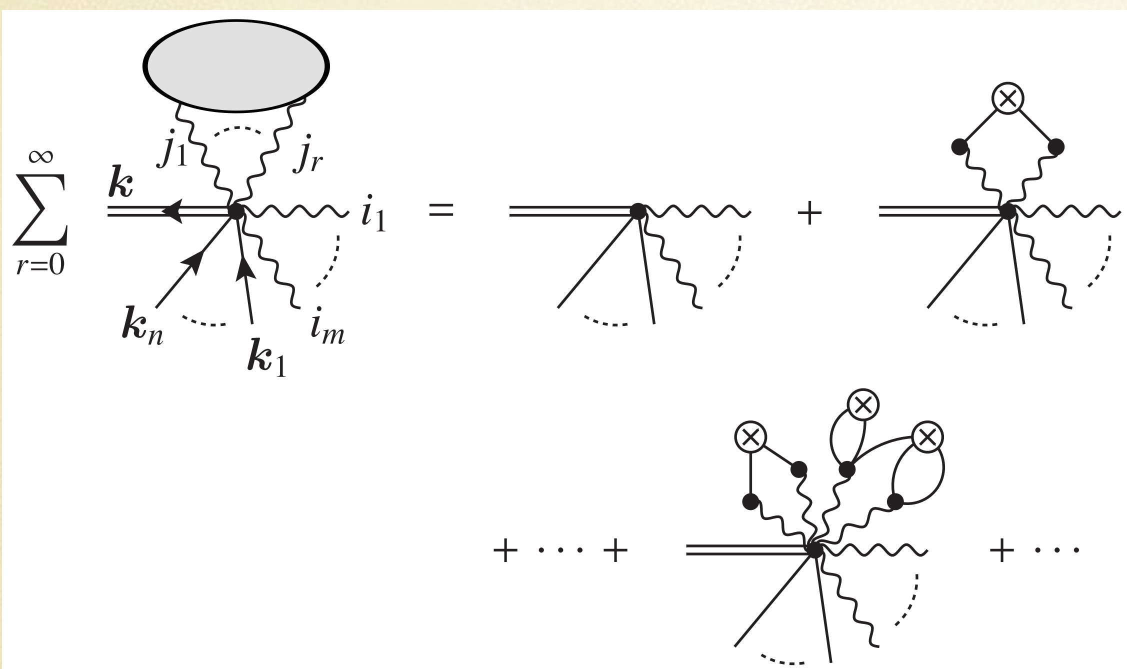


$$P_X(\boldsymbol{k}) = \left[\Gamma_X^{(1)}(\boldsymbol{k})\right]^2 P_{\rm L}(k) + \frac{1}{2}\int_{\boldsymbol{k}_{12}=\boldsymbol{k}} \left[\Gamma_X^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2)\right]^2 P_{\rm L}(k_1) P_{\rm L}(k_2) + \Gamma_X^{(1)}(\boldsymbol{k}) \int_{\boldsymbol{k}_{12}=\boldsymbol{k}} \Gamma_X^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2) B_{\rm L}(k, k_1, k_2),$$

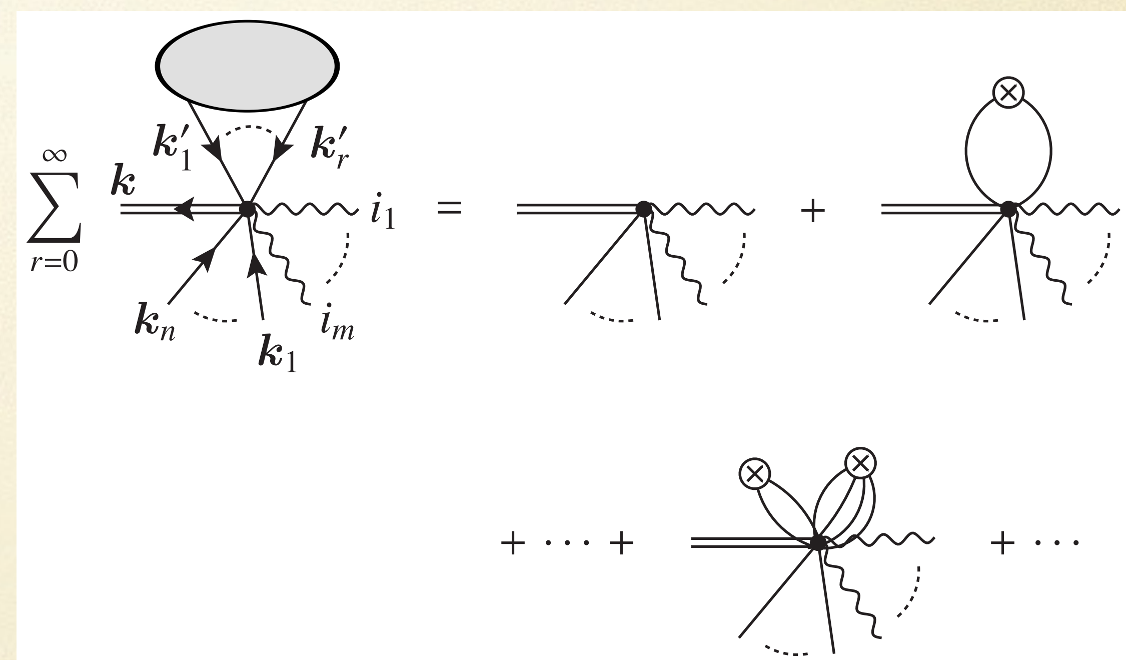
# Multi-point propagator

- Full evaluations of MP propagator are difficult
- Partial resummations in the Lagrangian PT

Lagrangian vertex resummation




Lagrangian bias renormalization

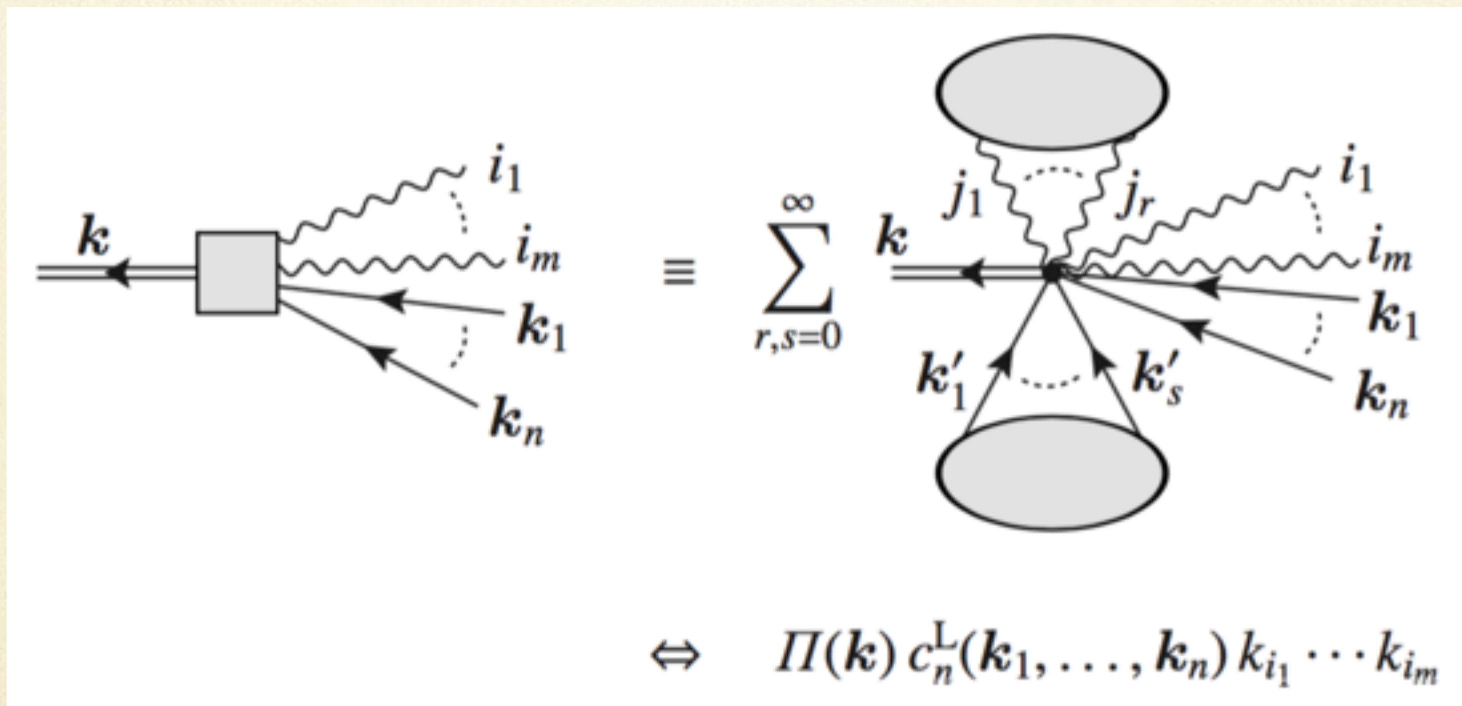



$$\Pi(\boldsymbol{k}) = \left\langle e^{-i\boldsymbol{k}\cdot\boldsymbol{\Psi}} \right\rangle$$
$$= \exp\left[\sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \left\langle (\boldsymbol{k}\cdot\boldsymbol{\Psi})^n \right\rangle_{\mathrm{c}}\right]$$

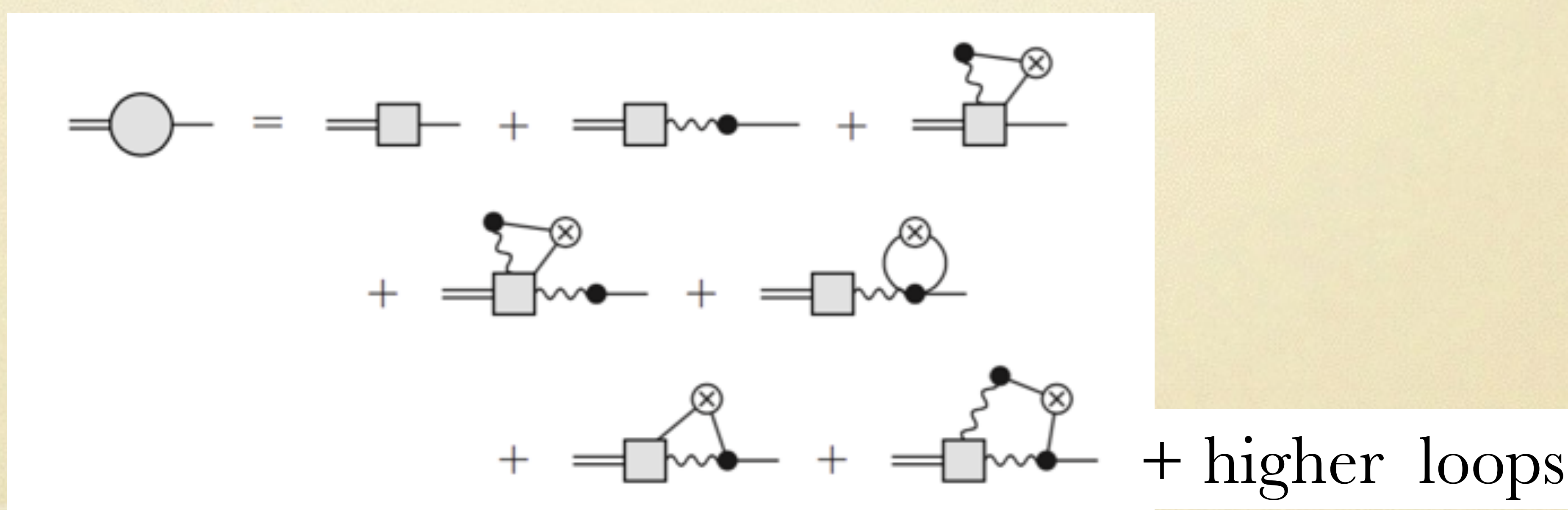
$$b_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n} \int \frac{d^3k'}{(2\pi)^3} \left. \frac{\delta^n \delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right|_{\delta_{\mathrm{L}}=0}$$
$$\Rightarrow\ c_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n} \int \frac{d^3k'}{(2\pi)^3} \left\langle \frac{\delta^n \delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right\rangle$$

# Vertex in iPT

- Combine the two resummations



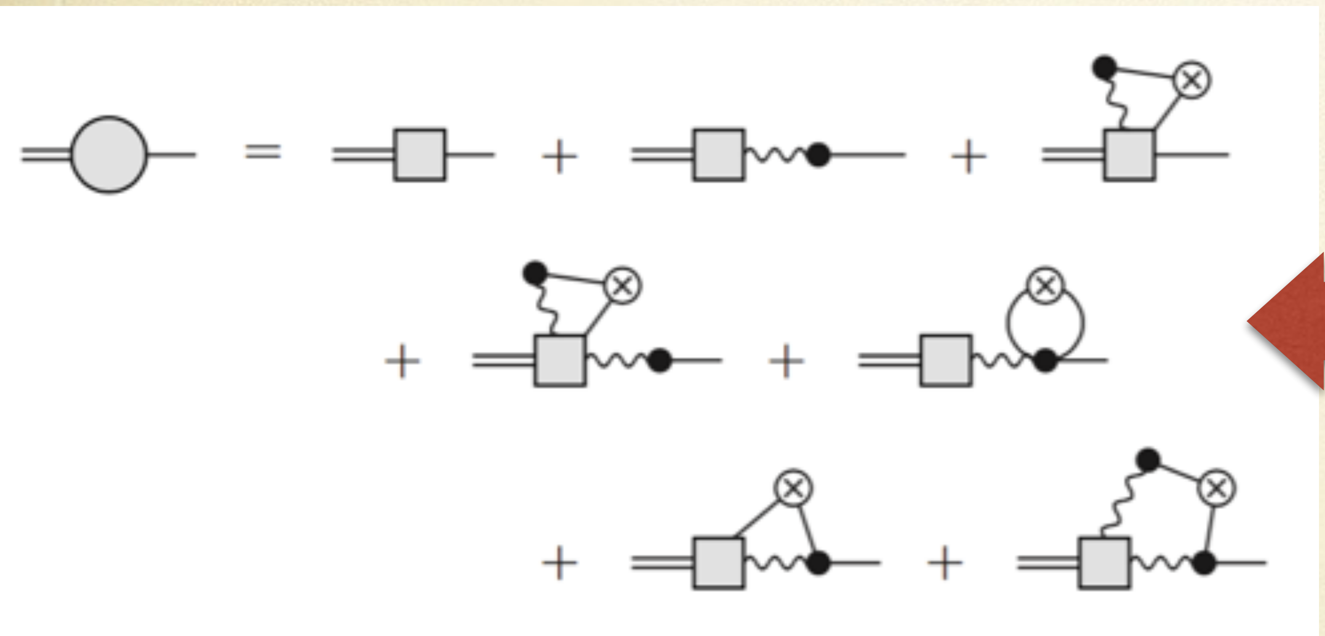

- Ex.)



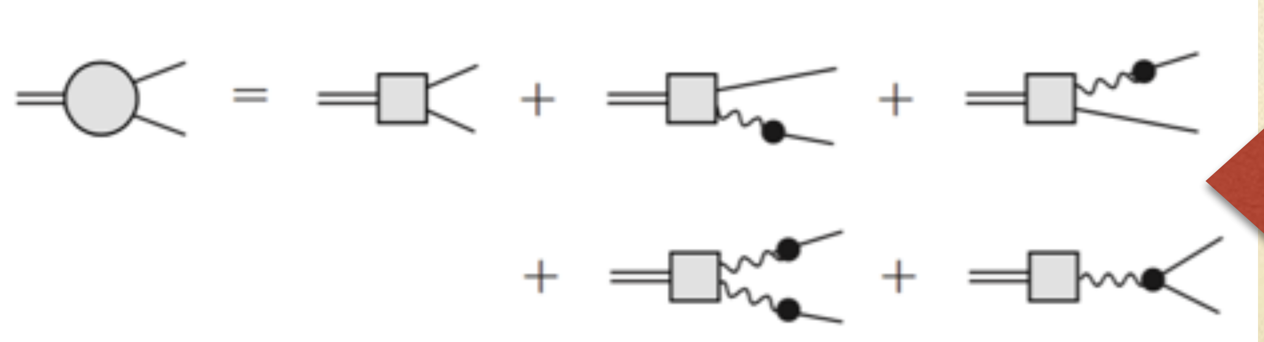
# One-loop propagators in iPT

$$\Gamma_X^{(n)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) = \Pi(\boldsymbol{k}_{1\cdots n}) \hat{\Gamma}_X^{(n)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n),$$

$$\Pi(\boldsymbol{k}) = \exp\left\{-\frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\left[\boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(\boldsymbol{p})\right]^2 P_{\mathrm{L}}(p)\right\},$$

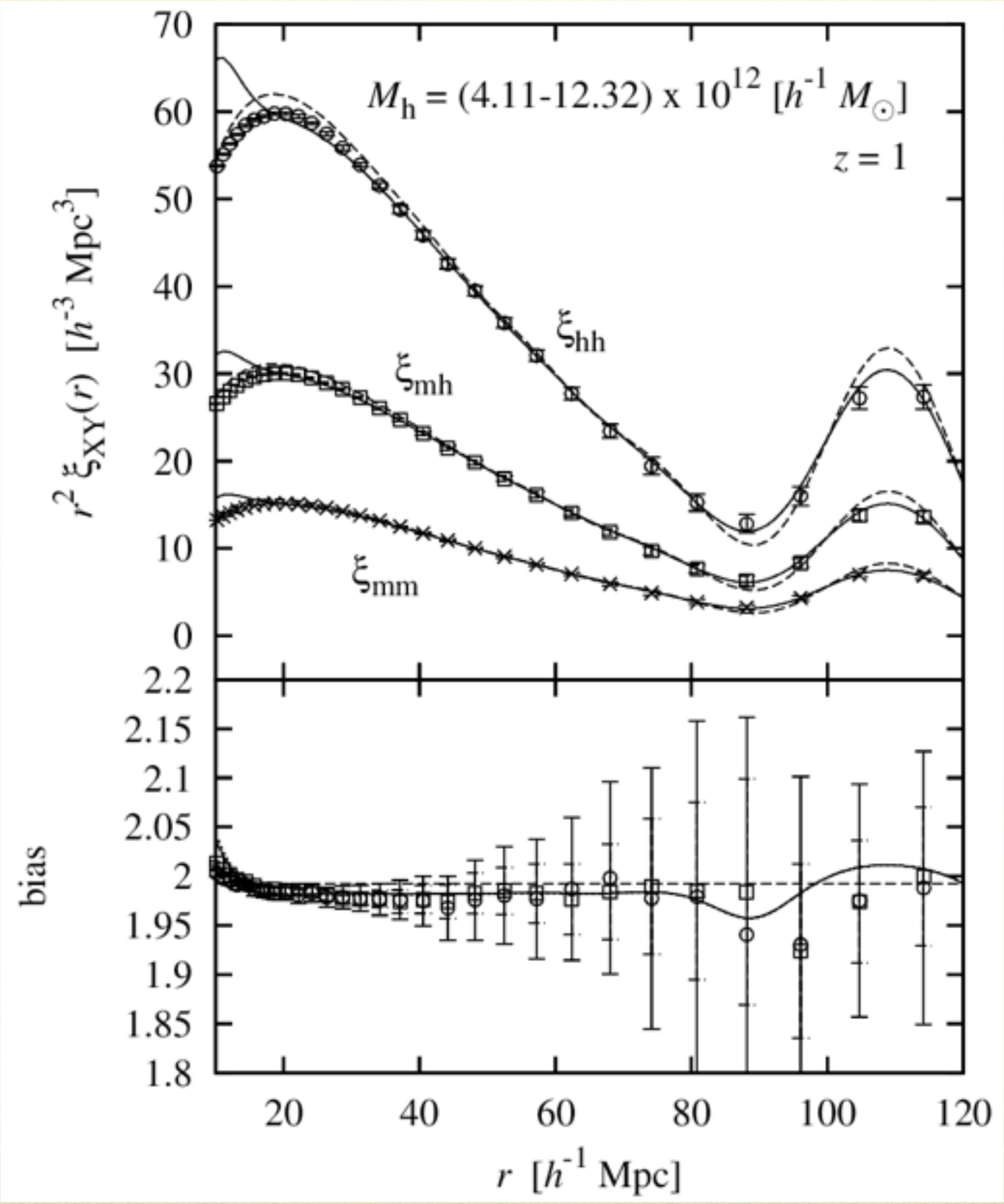


$$\begin{aligned}\hat{\Gamma}_X^{(1)}(\boldsymbol{k}) &= c_X^{(1)}(k) + \boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(\boldsymbol{k}) \\ &\quad + \int \frac{d^3p}{(2\pi)^3} P_{\mathrm{L}}(p)\Big\{c_X^{(2)}(\boldsymbol{k},\boldsymbol{p})\left[\boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(-\boldsymbol{p})\right] \\ &\quad + c_X^{(1)}(p)\left[\boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(-\boldsymbol{p})\right]\left[\boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(\boldsymbol{k})\right] \\ &\quad + \frac{1}{2}\boldsymbol{k}\cdot\boldsymbol{L}^{(3)}(\boldsymbol{k},\boldsymbol{p},-\boldsymbol{p}) \\ &\quad + c_X^{(1)}(p)\left[\boldsymbol{k}\cdot\boldsymbol{L}^{(2)}(\boldsymbol{k},-\boldsymbol{p})\right] \\ &\quad + \left[\boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(\boldsymbol{p})\right]\left[\boldsymbol{k}\cdot\boldsymbol{L}^{(2)}(\boldsymbol{k},-\boldsymbol{p})\right]\Big\},\end{aligned}$$



$$\begin{aligned}\hat{\Gamma}_X^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) &= c_X^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) + c_X^{(1)}(\boldsymbol{k}_1)\left[\boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(\boldsymbol{k}_2)\right] \\ &\quad + c_X^{(1)}(\boldsymbol{k}_2)\left[\boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(\boldsymbol{k}_1)\right] + \left[\boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(\boldsymbol{k}_1)\right]\left[\boldsymbol{k}\cdot\boldsymbol{L}^{(1)}(\boldsymbol{k}_2)\right] \\ &\quad + \boldsymbol{k}\cdot\boldsymbol{L}^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2),\end{aligned}$$

# Halo clustering: slight scale-dependence of bias around BAO



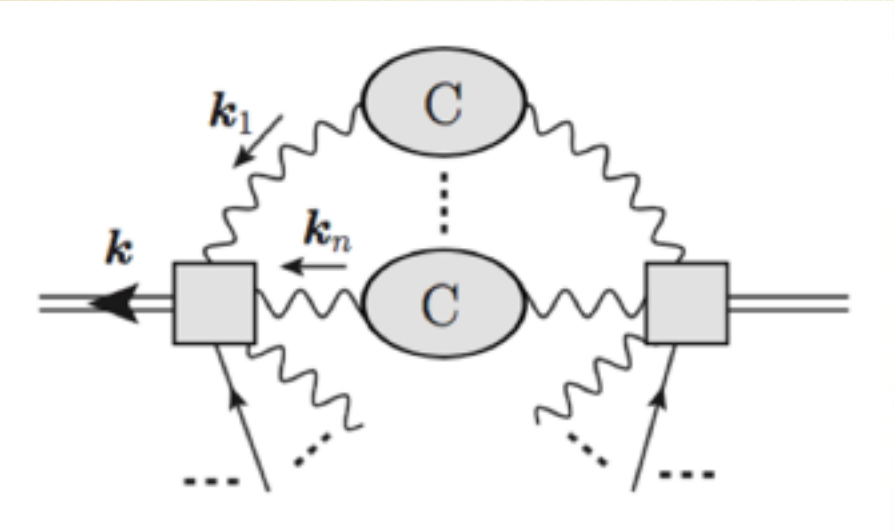

TM (2014)

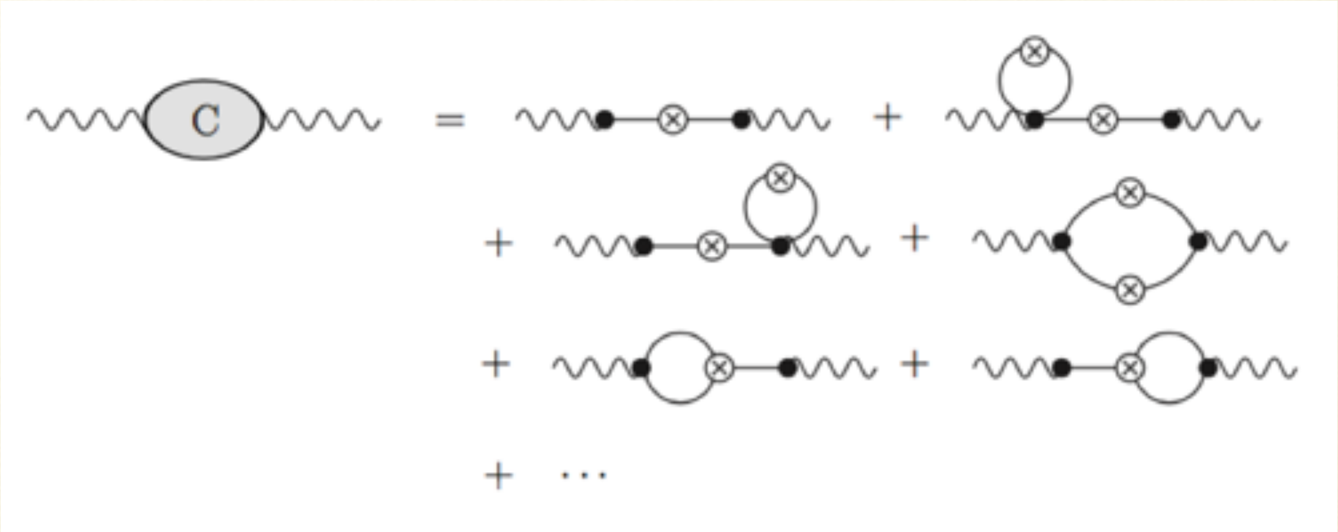
# CLPT

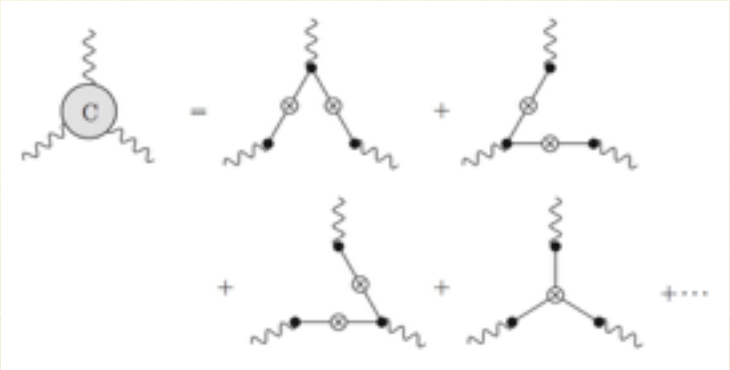
- **Convolution Lagrangian Perturbation Theory (CLPT)**

Carlson, Reid & White (2013);
Vlah, Seljuk & Baldauf (2014)

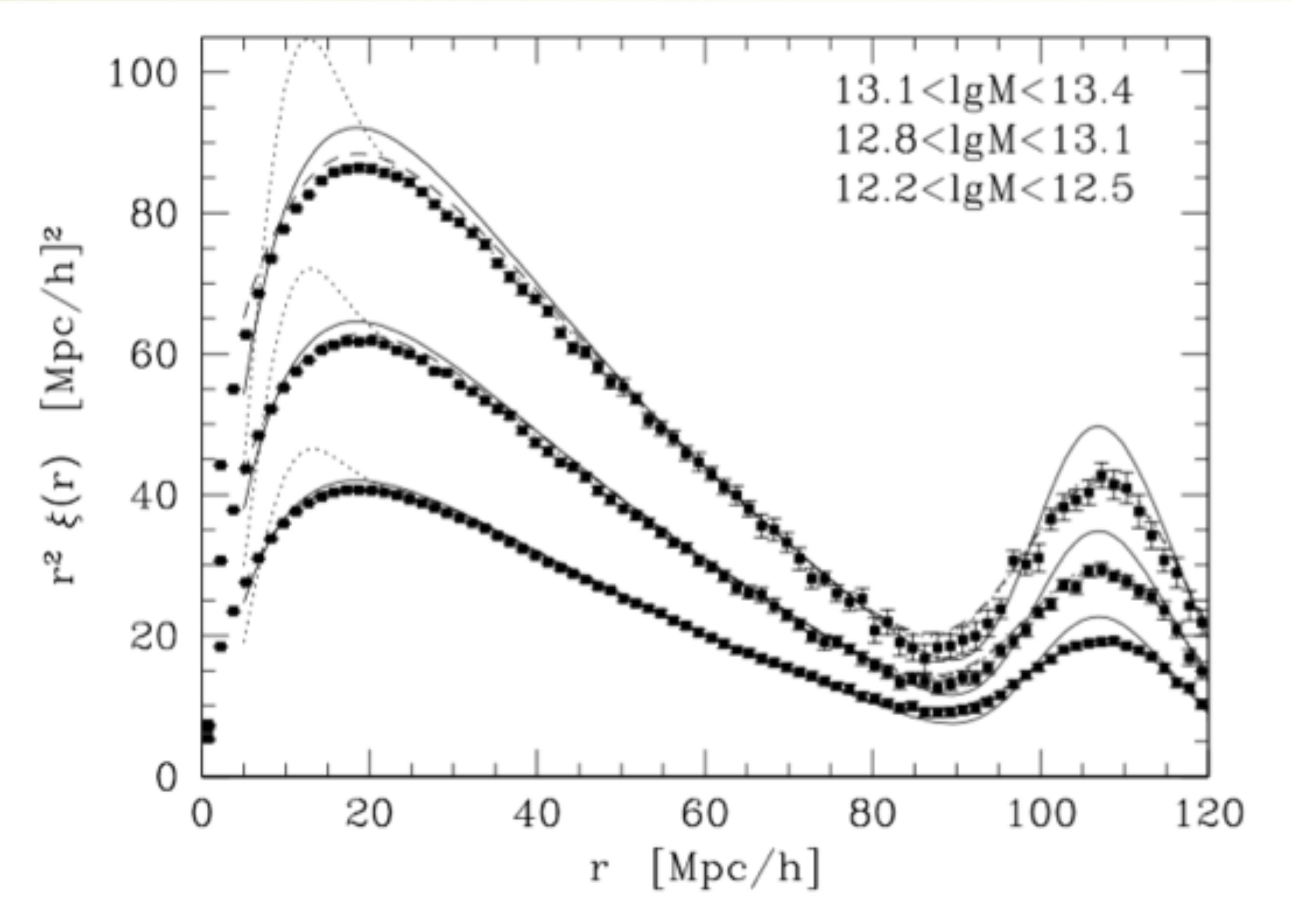
  - CLPT is an extension of the earlier version of iPT (LRT: Lagrangian Resummation Theory)
  - CLPT extension is also straightforwardly applied for iPT

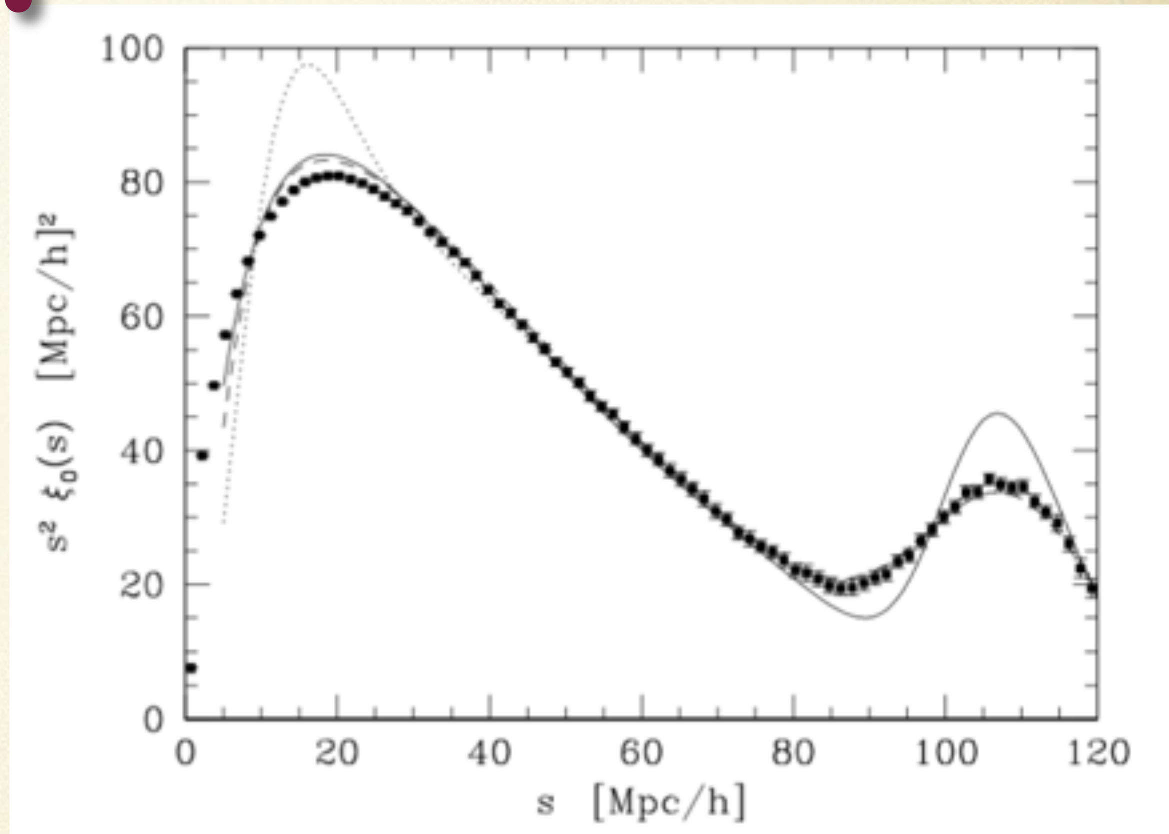


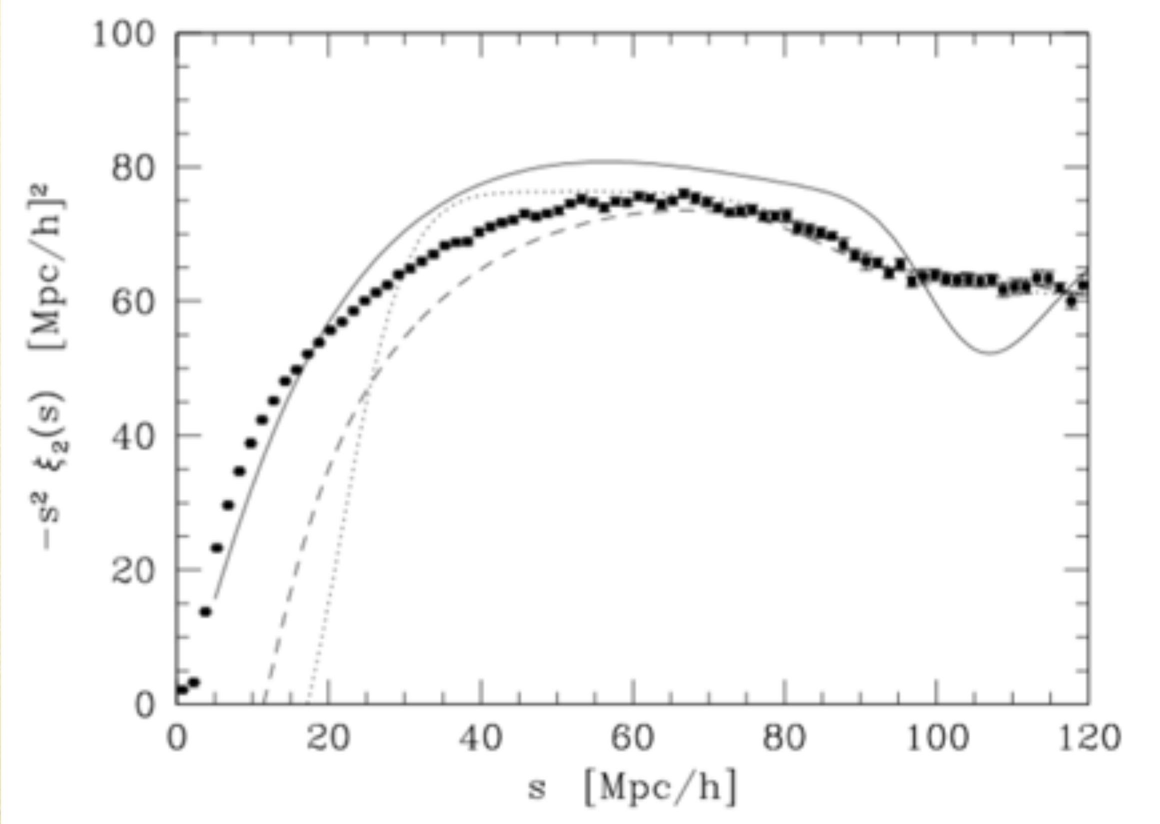



# CLPT




Carlson, Reid & White (2013)

# Impacts of biasing schemes

arXiv:1604.06579, with V. Desjacques

# Renormalized bias functions

- The "renormalized bias functions" is an essential piece in the iPT
  - Series of functions to characterize (nonlocal) biasing

$$\left\langle \frac{\delta^n \delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k})}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta_{\mathrm{D}}^3(\boldsymbol{k}_{1\cdots n}-\boldsymbol{k})c_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n).$$

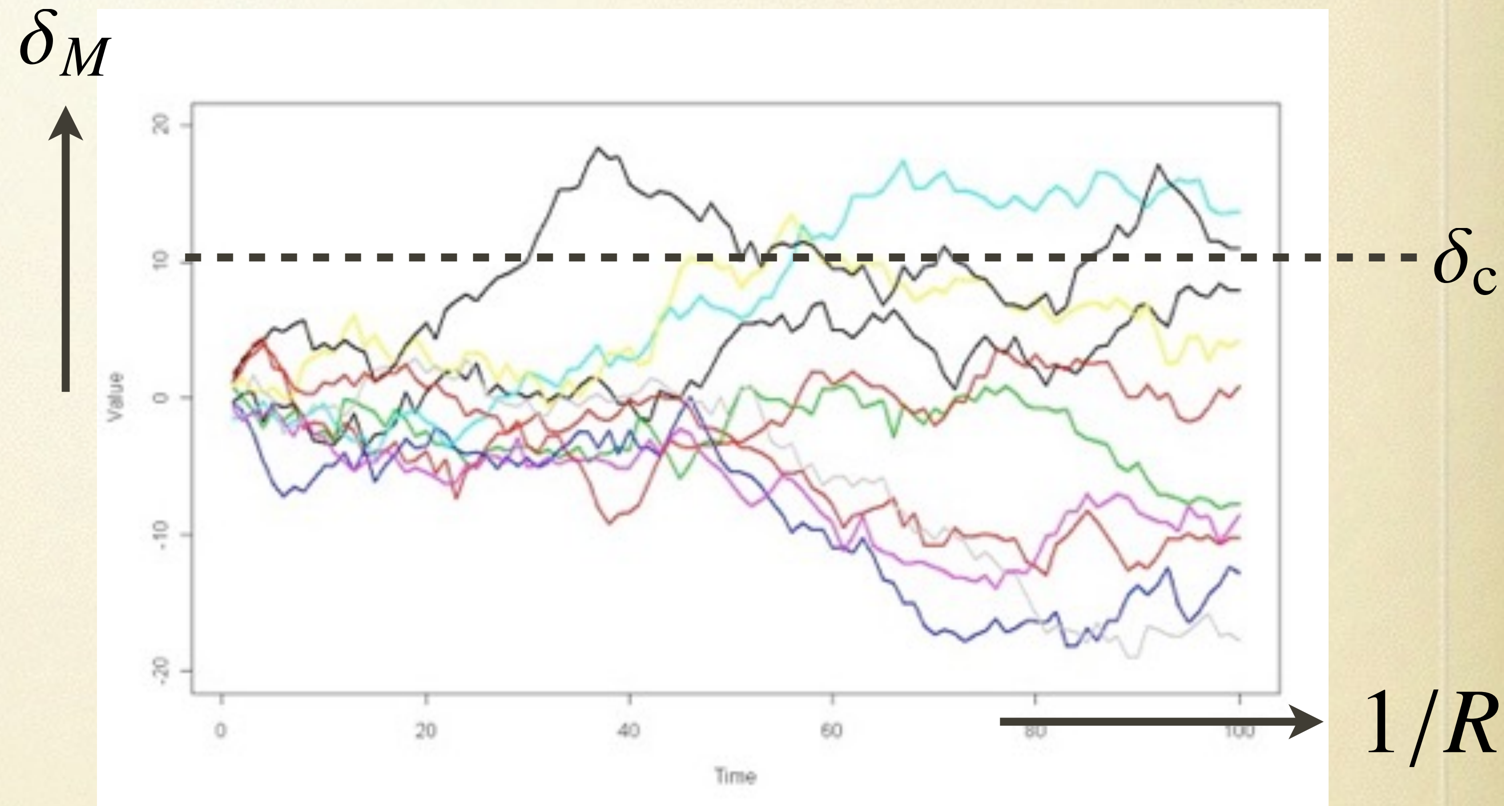
  - Counterpart of multi-point propagator for Lagrangian biasing

# Renormalized bias functions in "Halo model"

- Renormalized bias functions from Press-Schechter approach

$$n(\boldsymbol{x}, M) = -\frac{2\bar{\rho}_0}{M}\frac{\partial}{\partial M}\Theta\left[\delta_M(\boldsymbol{x}) - \delta_{\rm c}\right],$$

**"Localized" differential mass function**




$$c_1^{\rm L}(k) = b_1^{\rm L} W(kR) + \frac{1}{\delta_{\rm c}}\frac{\partial W(kR)}{\partial \ln \sigma_M}$$

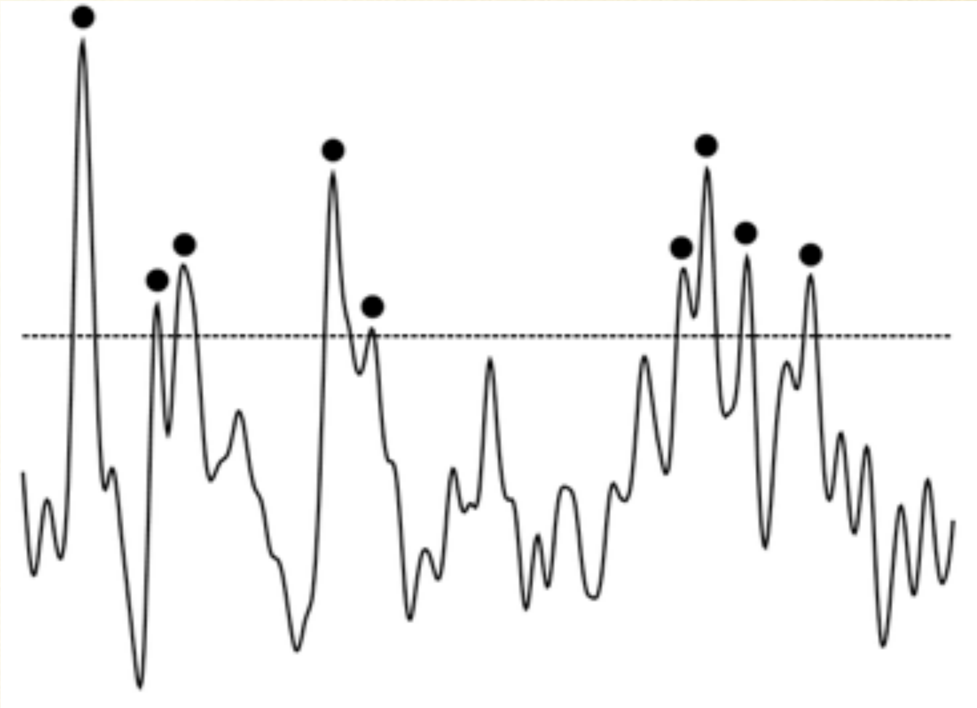
$$c_2^{\rm L}(\boldsymbol{k}_1, \boldsymbol{k}_2) = b_2^{\rm L} W(k_1 R) W(k_2 R) + \frac{\delta_{\rm c} b_1^{\rm L} + 1}{\delta_{\rm c}^2}\frac{\partial}{\partial \ln \sigma_M}\left[W(k_1 R) W(k_2 R)\right]$$

# Renormalized bias functions in "Peaks model"

$$n_{\rm pk} = \frac{3^{3/2}}{{R_*}^3}\delta_{\rm D}(\nu - \nu_{\rm c})\delta^3_{\rm D}(\boldsymbol{\eta})\Theta(\lambda_3)\,|\det\boldsymbol{\zeta}|\,,$$

**differential number density of peaks**

BBKS 1986



$$c_1^{\rm L}(k) = W(kR)\left[b_{10} + b_{11}k^2\right]$$

$$c_2^{\rm L}(\boldsymbol{k}_1, \boldsymbol{k}_2) = W(k_1R)W(k_2R)\Big\{b_{20} + b_{11}\left({k_1}^2 + {k_2}^2\right) + b_{02}{k_1}^2{k_2}^2 - 2\chi_1(\boldsymbol{k}_1\cdot\boldsymbol{k}_2) + \omega_{10}\left[3(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2 - {k_1}^2{k_2}^2\right]\Big\}$$

$$b_{ij} \equiv \frac{1}{{\sigma_0}^i{\sigma_2}^j\bar{n}_{\rm pk}}\int d^{10}y\, n_{\rm pk}H_{ij}(\nu, J_1)\mathcal{P},$$

$$\chi_k \equiv \frac{(-1)^k}{{\sigma_1}^{2k}\bar{n}_{\rm pk}}\int d^{10}y\, n_{\rm pk}L_k^{(1/2)}\left(\frac{3}{2}\eta^2\right)\mathcal{P},$$

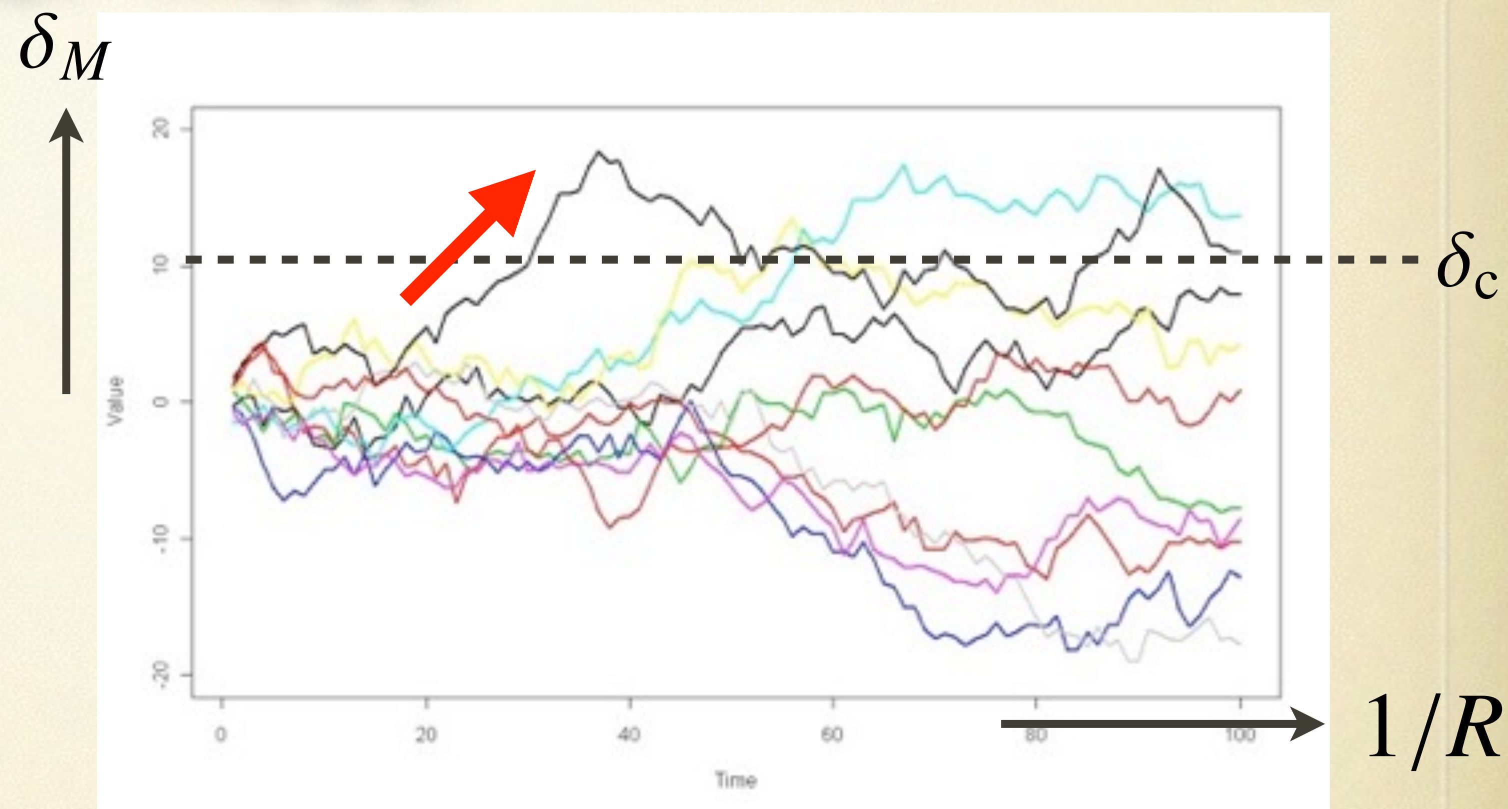
$$\omega_{l0} \equiv \frac{(-1)^l}{{\sigma_2}^{2l}\bar{n}_{\rm pk}}\int d^{10}y\, n_{\rm pk}L_l^{(3/2)}\left(\frac{5}{2}J_2\right)\mathcal{P}.$$

Desjacques+ 2013, Lazeyras+ 2015

# Renormalized bias functions in "ESP model"

Excursion set theory
+ Peak constraints
+ Upcrossing constraint

Appel&Jones 1990, Desjacques 2013, Paranjape&Sheth 2013, Biagetti+ 2014,...




$$n_{\rm ESP} = -\left(\frac{d\sigma_{s0}}{dR_s}\right)^{-1} \varDelta_{s0}\frac{\mu_s}{\nu_s}\Theta(\mu_s)\, n_{\rm pk},$$

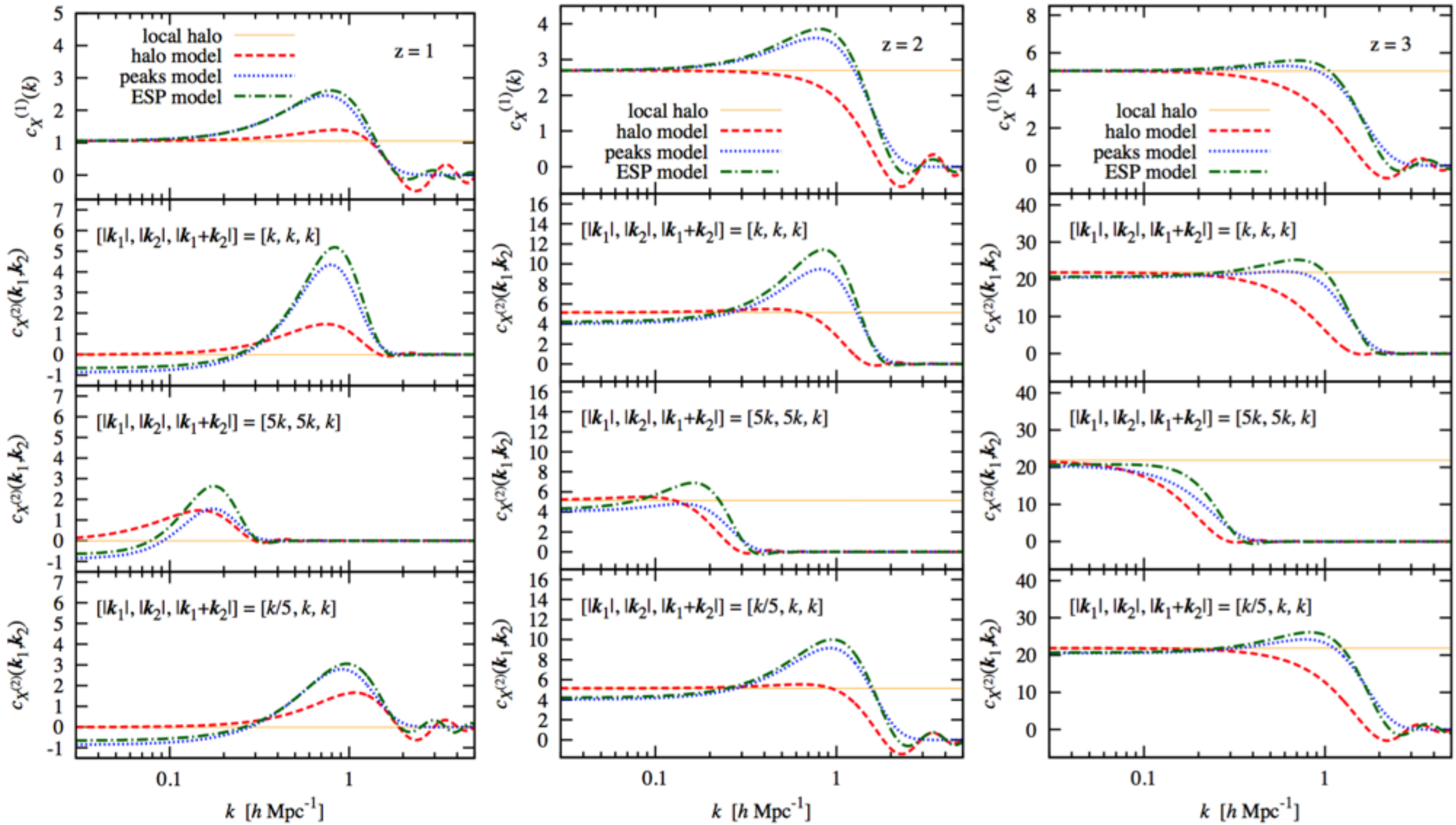
$$\mu_s = -\frac{1}{\varDelta_{s0}}\frac{\partial\delta_s}{\partial R_s}, \quad \varDelta_{s0} = \left\langle\left(\frac{\partial\delta_s}{\partial R_s}\right)^2\right\rangle^{1/2}.$$

$$c_X^{(1)}(k) = b_{100}W(kR) + b_{010}k^2\bar{W}(k\bar{R}) - b_{001}kW'(kR),$$

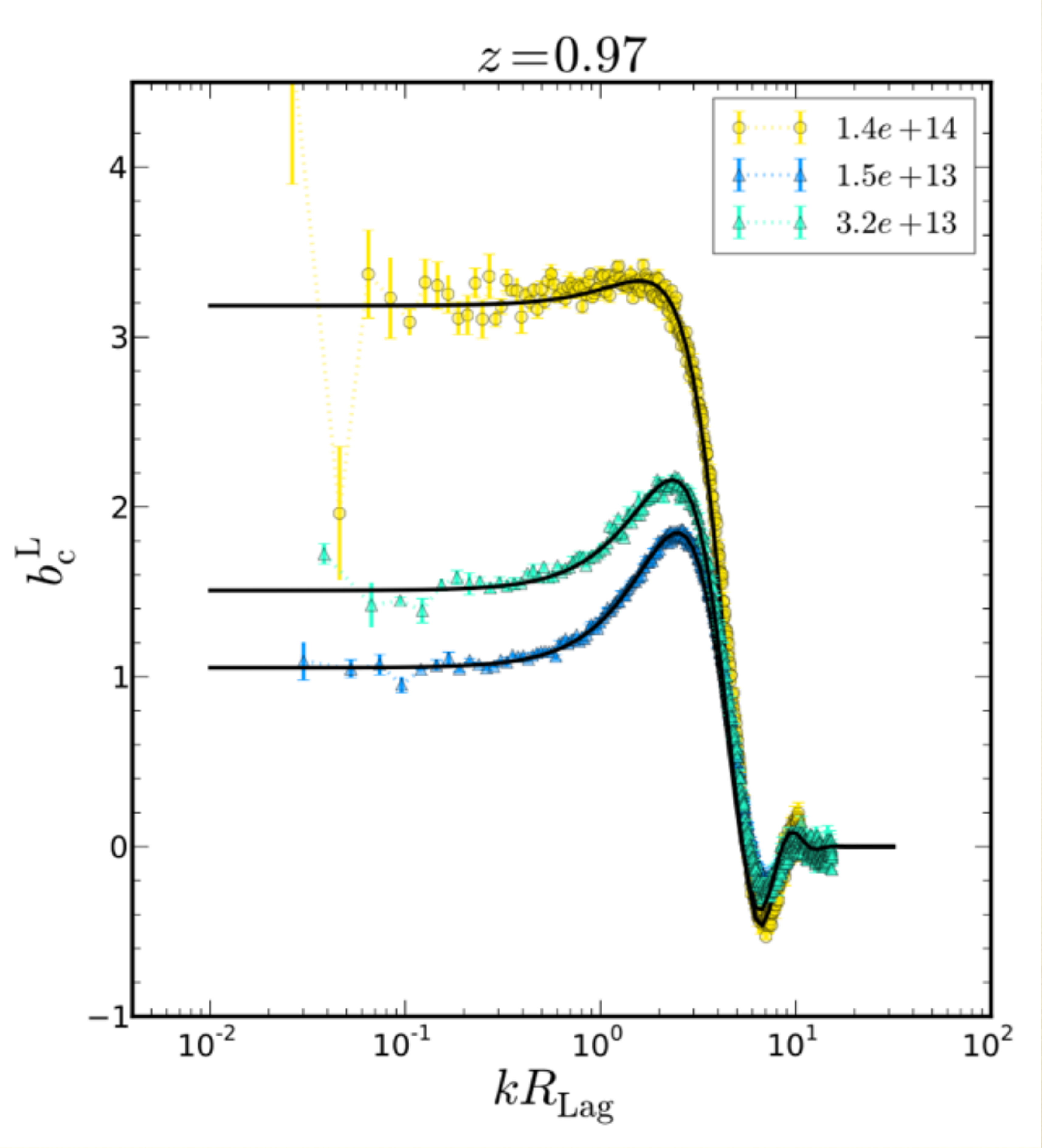
$$\begin{aligned} c_X^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) = {}& b_{200}W(k_1R)W(k_2R) + b_{110}\left[{k_2}^2W(k_1R)\bar{W}(k_2\bar{R}) + (1\leftrightarrow 2)\right] \\ &+ \left\{b_{020}{k_1}^2{k_2}^2 + \omega_{10}\left[3(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)^2 - {k_1}^2{k_2}^2\right] - 2\chi_1(\boldsymbol{k}_1\cdot\boldsymbol{k}_2)\right\}\bar{W}(k_1\bar{R})\bar{W}(k_2\bar{R}) \\ &- b_{101}\left[k_1W'(k_1R)W(k_2R) + (1\leftrightarrow 2)\right] - b_{011}\left[k_1{k_2}^2W'(k_1R)\bar{W}(k_2\bar{R}) + (1\leftrightarrow 2)\right] + b_{002}k_1k_2W'(k_1R)W'(k_2R), \end{aligned}$$

Dizgah+ 2015

# Renormalized bias functions

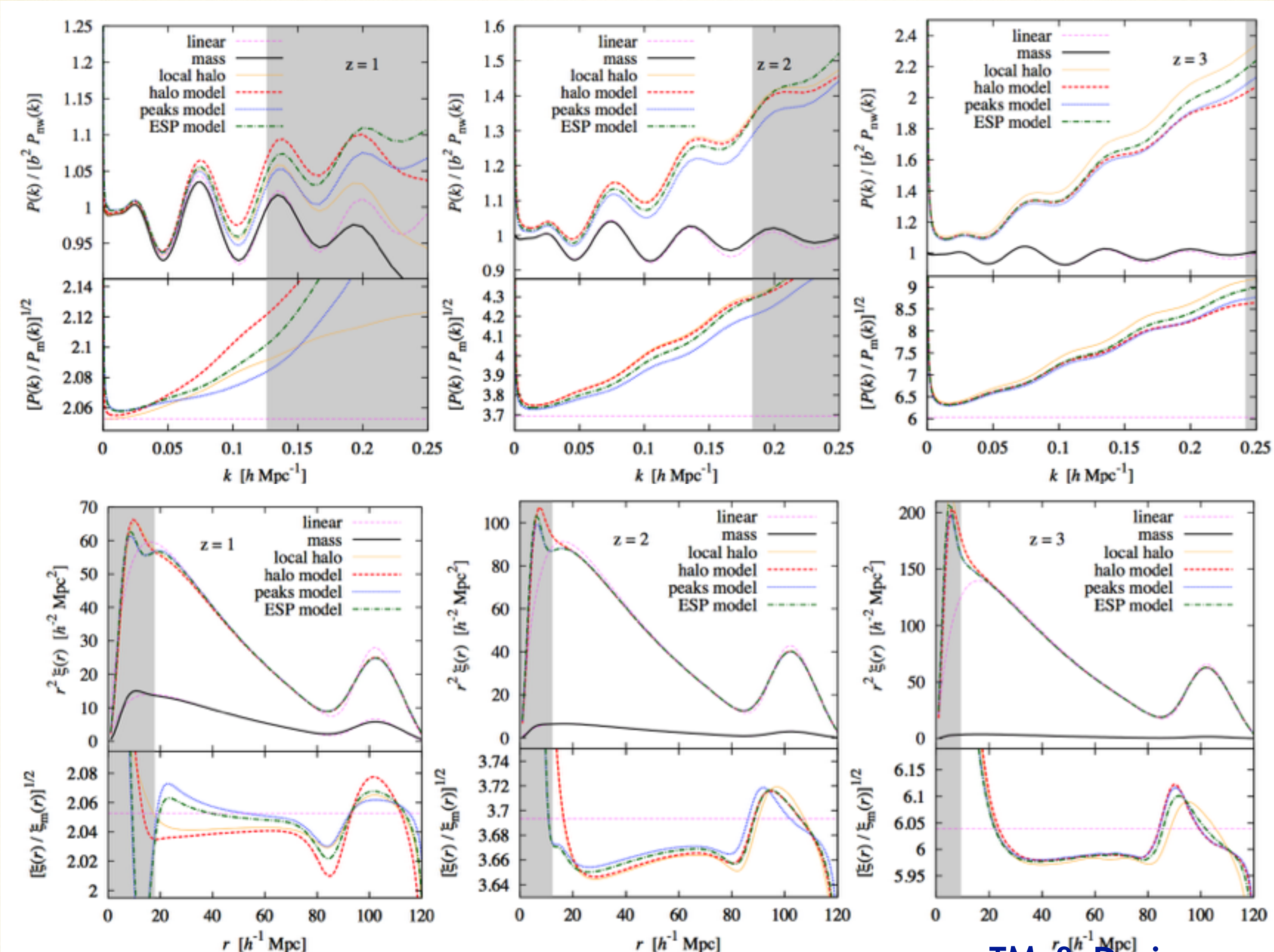



TM & Desjacques (2016)



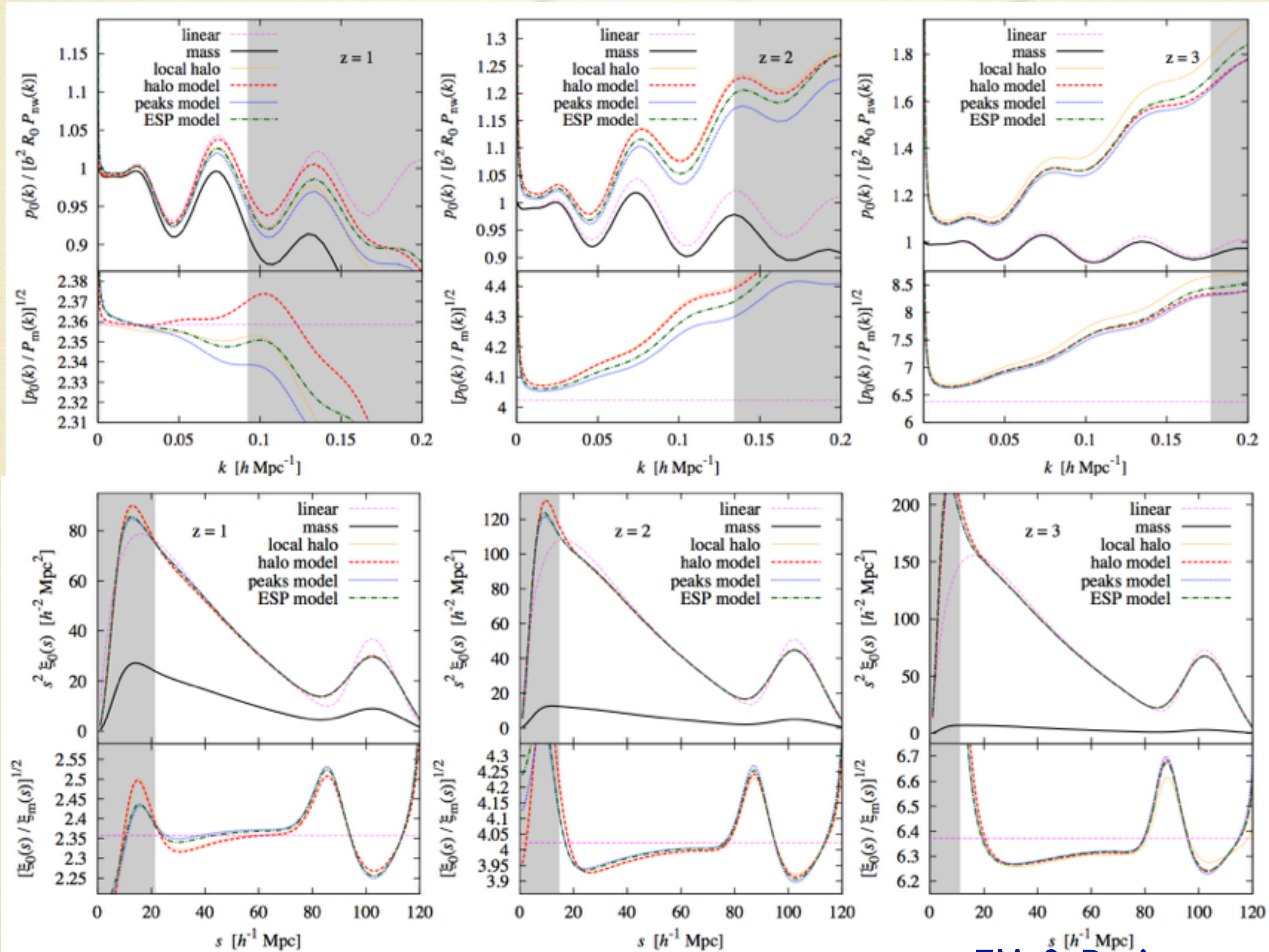

Chan+ (2016)

# Power spectra & correlation functions



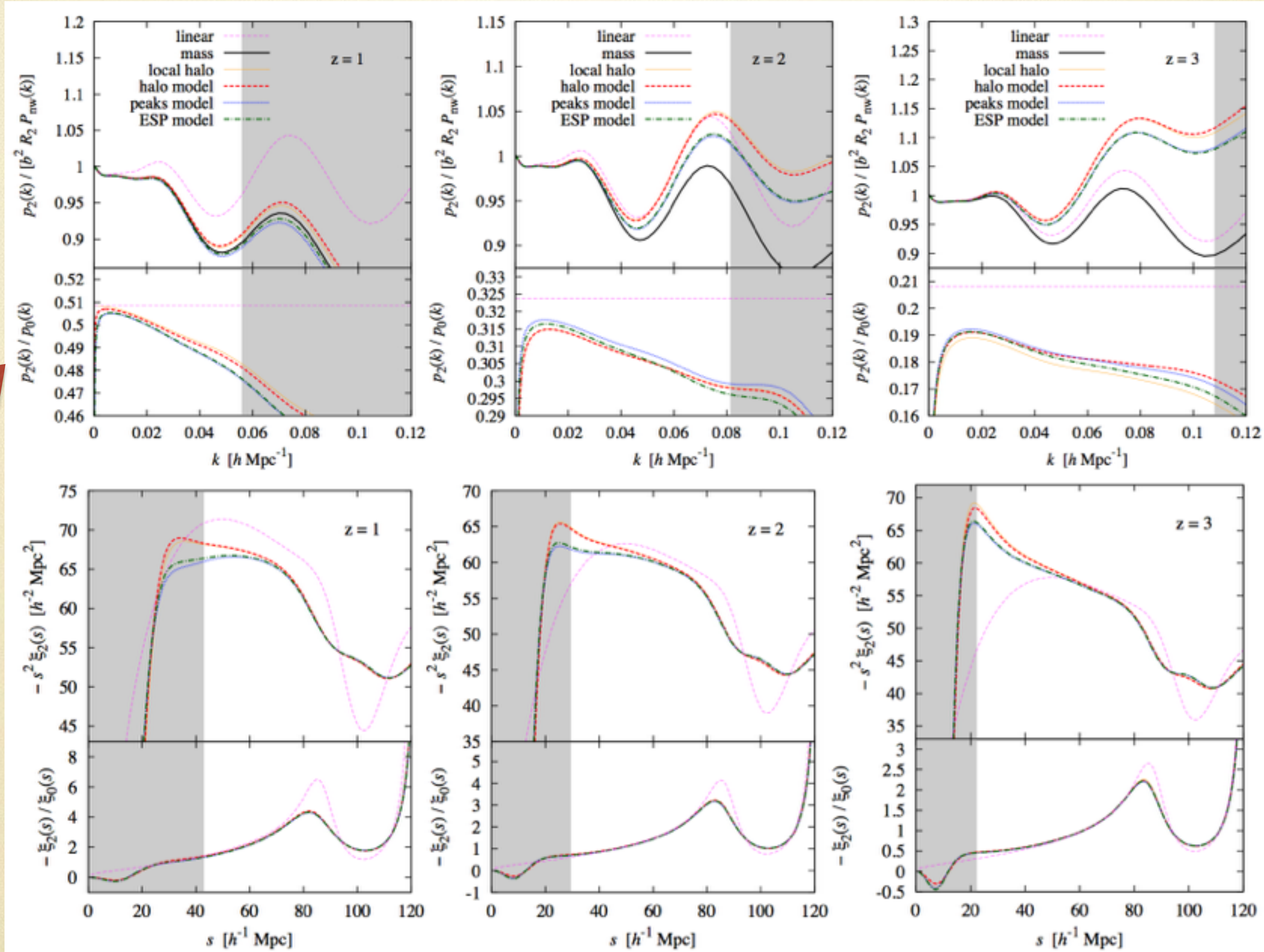

TM & Desjacques (2016)

# Redshift space, monopole




TM & Desjacques (2016)

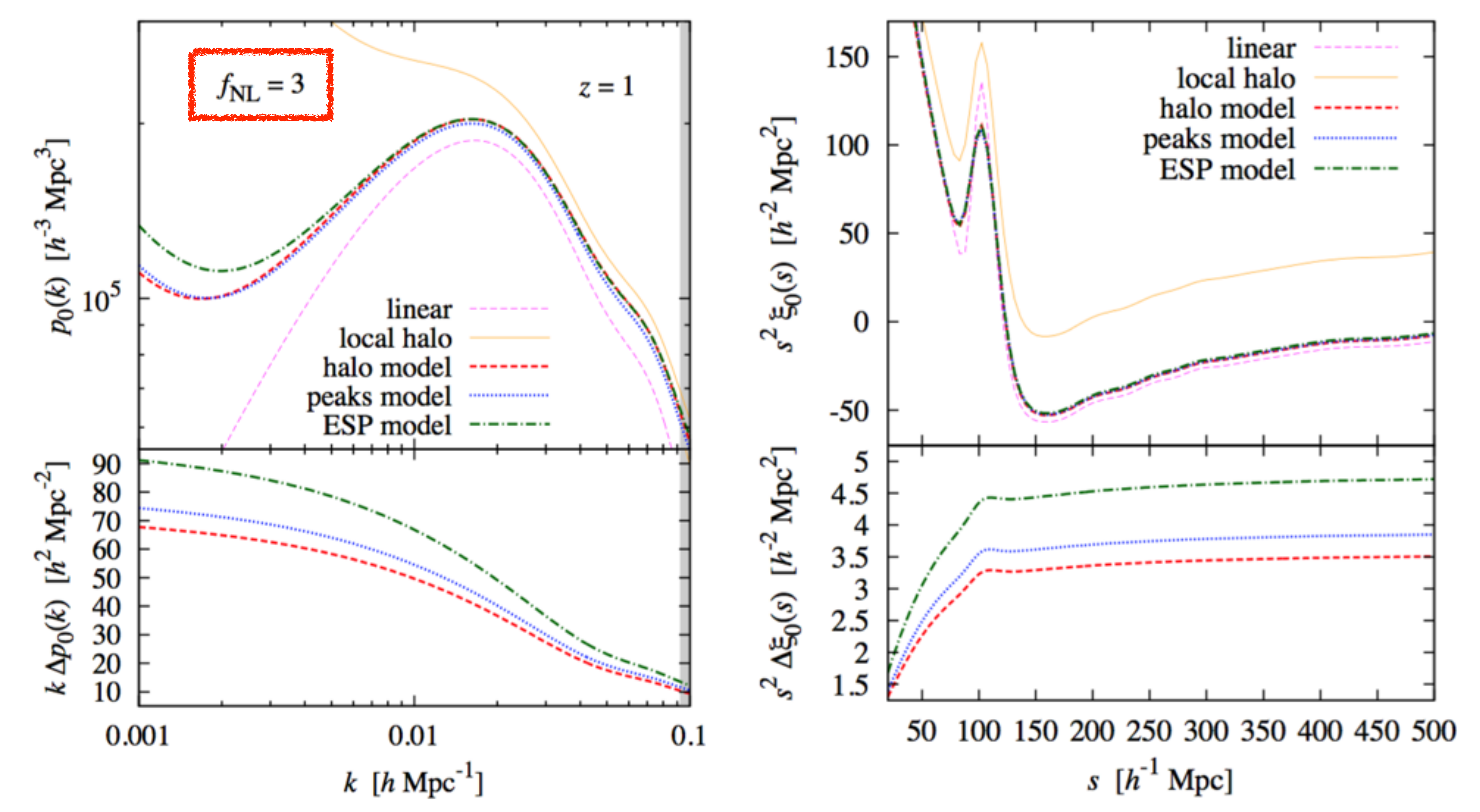
# Redshift-space distortions, quadrupole




$f\,\sigma_8$

TM & Desjacques (2016)

# Primordial non-Gaussianity




TM & Desjacques (2016)

# Summary

- Dependence on the bias models in weakly nonlinear regime
  - 2-4% for the power spectrum
  - < 1% for the correlation function
- Still important effects for the precision cosmology