Massive Nνtrinos in NL LSS: A Consistent Perturbation Theory

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NL LSS: Theory Meets Expectations
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In short: High precision

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4. Well-defined perturbative framework
5. Computational efficiency for practical use

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6. + Non-crucial ingredient: Exact evaluation of NL ν perturbation

The Equations of Backreaction Correction

The equations for the CDM+baryons component

$$
\begin{aligned}
\dot{\delta}_c + \frac{1}{a}\partial_j\left[(1+\delta_c)v_c^j\right] &= 0, \\
\dot{v}_c^i + Hv_c^i + \frac{1}{a}v_c^j\partial_j v_c^i + \frac{1}{a}\partial^i\phi &= -\frac{1}{a}c_s^2\partial^i\delta_c, \\
\nabla^2\phi &= \frac{3}{2}\frac{\Omega_m^0 H_0^2}{a}\left[(1-f_\nu)\,\delta_c + f_\nu\delta_\nu\right].
\end{aligned}
$$

f_ν as a coupling constant

$$
\begin{aligned}
\delta^c &\equiv \textstyle\sum_{i=1}^{\infty}\delta_i + f_\nu\sum_{i=1}^{\infty}\tilde{\delta}_i \equiv \underbrace{\delta}_{\text{“bare”}} + f_\nu \underbrace{\tilde{\delta}}_{\text{BR correction}}, \\
\theta^c &\equiv \textstyle\sum_{i=1}^{\infty}\theta_i + f_\nu\sum_{i=1}^{\infty}\tilde{\theta}_i \equiv \theta + f_\nu \tilde{\theta}, \\
\delta^\nu &\equiv \textstyle\sum_{i=1}^{\infty}\delta_i^\nu .
\end{aligned}
$$

The Equations of Backreaction Correction

The bare equations for the baseline massless ν cosmology

$$
\begin{aligned}
a\dot{\delta} + \theta &= -\frac{1}{(2\pi)^3}\int d^3p\left[\alpha(\vec{p},\vec{k}-\vec{p})\delta(\vec{k}-\vec{p})\theta\left(\vec{p}\right)\right],\\
a\dot{\theta} + aH\theta + \frac{3}{2}\frac{\Omega_m^0 H_0^2}{a}\delta &= -\frac{1}{(2\pi)^3}\int d^3p\left[\beta(\vec{p},\vec{k}-\vec{p})\theta(\vec{k}-\vec{p})\theta(\vec{p})\right] + c_s^2 k^2\delta,
\end{aligned}
$$

$$
\alpha(\vec{p},\vec{q}) = \frac{(\vec{p}+\vec{q})\cdot\vec{p}}{p^2}, \qquad \beta(\vec{p},\vec{q}) = \frac{1}{2}\frac{(\vec{p}+\vec{q})^2\,\vec{p}\cdot\vec{q}}{p^2 q^2}.
$$

The Equations of Backreaction Correction

The backreaction correction equations

$$a\dot{\tilde{\delta}} + \tilde{\theta} = -\frac{1}{(2\pi)^3}\int d^3p \left[\alpha(\vec{p},\vec{k}-\vec{p})\left(\tilde{\delta}(\vec{k}-\vec{p})\theta\left(\vec{p}\right) + \delta(\vec{k}-\vec{p})\tilde{\theta}\left(\vec{p}\right)\right)\right],$$

$$a\dot{\tilde{\theta}} + aH\tilde{\theta} + \frac{3}{2}\frac{\Omega_m^0 H_0^2}{a}\left(\tilde{\delta} + \underbrace{\delta^\nu - \delta}_{\text{Anti-Gravity}}\right) = -\frac{2}{(2\pi)^3}\int d^3p \left[\beta(\vec{p},\vec{k}-\vec{p})\theta(\vec{k}-\vec{p})\tilde{\theta}(\vec{p})\right] + c_s^2 k^2\tilde{\delta}.$$

The Equations of Backreaction Correction

Nonhomogeneous linear equation due to anti-Gravity source

$$-a^4 H^2 \tilde{\delta}'' - (3a^3 H^2 + a^4 H H')\tilde{\delta}' + \frac{3}{2}\frac{\Omega_m^0 H_0^2}{a}\tilde{\delta} = \frac{3}{2}\frac{\Omega_m^0 H_0^2}{a}(\delta - \delta^\nu)$$

Green's function for any f_ν

$$G(a,\bar{a}) = \frac{H(a)}{\bar{a}}\int_a^{\bar{a}} dx \frac{1}{x^3 H^3(x)}\,\theta_H(a-\bar{a})$$

Leading Backreaction Correction

Linear solution

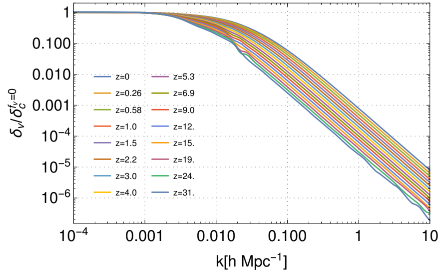
$$\tilde{\delta}_1(a,\vec{k}) = \frac{3}{2}\Omega_m^0 H_0^2 \int_{a_{in}}^{a} d\bar{a}\, \frac{G(a,\bar{a})}{\bar{a}} \left[\delta_1(\bar{a},\vec{k}) - \delta_1^{\nu}(\bar{a},\vec{k})\right] + \tilde{\delta}_{in}(\vec{k}) - \tilde{\delta}'_{in}(\vec{k}) a_{in}^4 H^2(a_{in}) G(a,a_{in}),$$

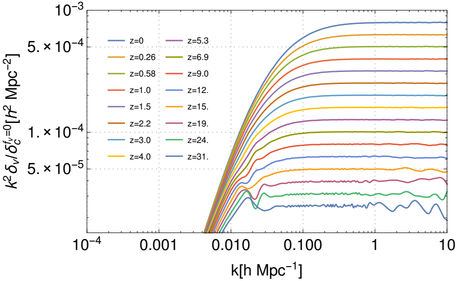
with initial conditions:

$$\tilde{\delta}_1(a_{in},\vec{k}) = \tilde{\delta}_{in}(\vec{k}), \qquad \partial_a \tilde{\delta}_1(a_{in},\vec{k}) = \tilde{\delta}'_{in}(\vec{k}).$$

Leading Backreaction Correction

The linear neutrino perturbation

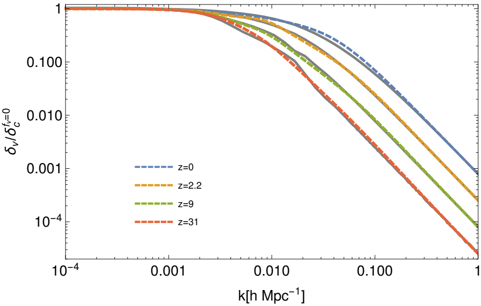




$$\frac{\delta^\nu(a,\vec{k})}{\delta(a,\vec{k})} \simeq \begin{cases} 1, & k \ll k_{nr}, \\ C\,a\left(\dfrac{f_\nu}{k}\right)^2, & k \gg k_{nr}. \end{cases}$$

Leading Backreaction Correction

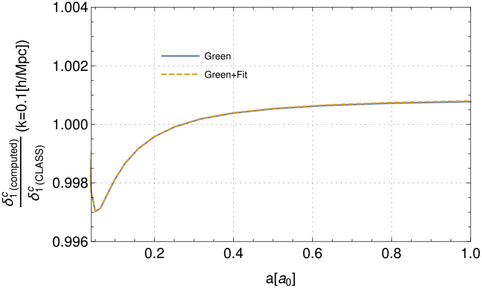
Representing the linear neutrino perturbation



$$\frac{\delta^\nu(a,\vec{k})}{\delta(a,\vec{k})} = \sum_{i=0}^{n} F_i\left(\left(\frac{f_\nu}{k}\right)^2\right)\, a^i, \quad n \in N$$

Leading Backreaction Correction

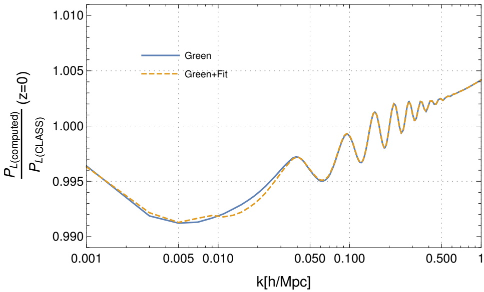
CDM+baryons linear density evolution check



$$\tilde{\delta}_1(a,\vec{k}) = \tilde{I}_{1,0}(a)\delta_1(\vec{k})[1-F_0(\vec{k})] - \tilde{I}_{1,j}(a)\delta_1(\vec{k})F_j(\vec{k})$$
$$+\tilde{\delta}_{in}(\vec{k}) - \tilde{\delta}'_{in}(\vec{k})a^4_{in}H^2(a_{in})G(a,a_{in}),$$
$$\tilde{I}_{1,j}(a) = \frac{3}{2}\Omega^0_m H^2_0 \int_{a_{in}}^{a} d\bar{a}\, G(a,\bar{a})D_+(\bar{a})\,\bar{a}^{j-1}.$$

Leading Backreaction Correction

Linear power spectrum check

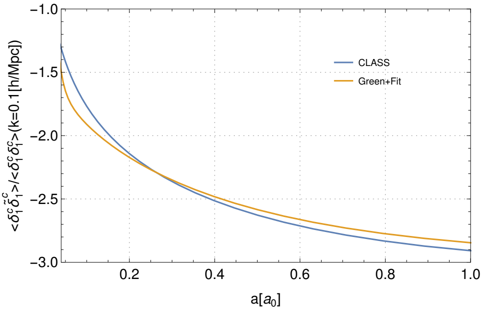


$$\delta_1^m \simeq \delta_1 - f_\nu \left(\delta_1 - \tilde{\delta}_1 - \delta_1^\nu \right) \Rightarrow$$

$$\langle \delta_1^m \, \delta_1^m \rangle \simeq (1 - 2f_\nu) \langle \delta_1 \, \delta_1 \rangle + \underbrace{2f_\nu \langle \delta_1 \, \delta_1^\nu \rangle}_{\text{FS of } \nu\text{s}} + \underbrace{2f_\nu \langle \delta_1 \, \tilde{\delta}_1 \rangle}_{\text{BR of CDM+b}} \,,$$

Leading Backreaction Correction

The relative effect, to be multiplied by $2f_\nu$, of the leading CDM+baryons backreaction



Leading Order Matter Bispectrum

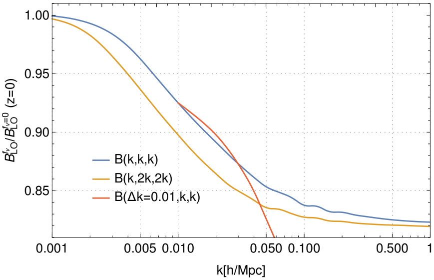
$$\langle\delta_1^m(\vec{k}_1)\,\delta_1^m(\vec{k}_2)\,\delta_2^m(\vec{k}_3)\rangle \simeq \langle\delta_1(\vec{k}_1)\,\delta_1(\vec{k}_2)\,\delta_2(\vec{k}_3)\rangle\,(1-3f_\nu)$$
$$+ f_\nu \underbrace{\left(\langle\delta_1^\nu(\vec{k}_1)\,\delta_1(\vec{k}_2)\,\delta_2(\vec{k}_3)\rangle + \langle\delta_1(\vec{k}_1)\,\delta_1^\nu(\vec{k}_2)\,\delta_2(\vec{k}_3)\rangle + \frac{\delta_1^\nu(k_3)}{\delta_1(k_3)}\langle\delta_1(\vec{k}_1)\,\delta_1(\vec{k}_2)\,\delta_2(\vec{k}_3)\rangle\right)}_{\nu\text{s free streaming}}$$
$$+ f_\nu \underbrace{\left(\langle\tilde{\delta}_1(\vec{k}_1)\,\delta_1(\vec{k}_2)\,\delta_2(\vec{k}_3)\rangle + \langle\delta_1(\vec{k}_1)\,\tilde{\delta}_1(\vec{k}_2)\,\delta_2(\vec{k}_3)\rangle + \langle\delta_1(\vec{k}_1)\,\delta_1(\vec{k}_2)\,\tilde{\delta}_2(\vec{k}_3)\rangle\right)}_{\text{CDM+baryons backreaction}},$$

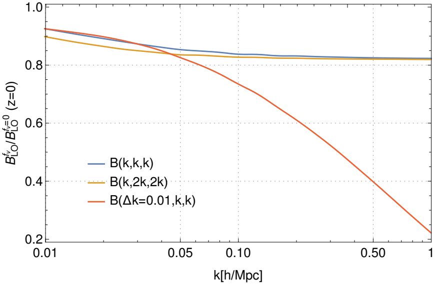
using

$$\delta^\nu \simeq \left(\frac{\delta_L^\nu}{\delta_L^c}\right)\delta^c.$$

Leading Order Matter Bispectrum

Ratio of LO matter bispectra with $f_\nu = 0.01291$ to $f_\nu = 0$





$$\frac{\Delta B_{LO}^{f_\nu}(k,k,k)}{B_{LO}^{f_\nu=0}(k,k,k)} \simeq -13.5 f_\nu$$

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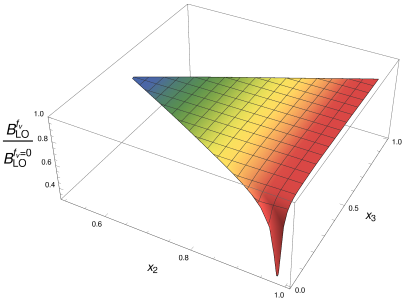

The shape dependence shows a suppression of similar values to the equilateral configuration suppression of $\sim -13.5 f_\nu$, whereas a steep enhanced suppression appears around the squeezed limit at high k modes.


thank you
danke
рахмат
謝謝
ngiyabonga
teşekkür ederim
gracias
merci
obrigado
bedankt
dziękuję
sukriya
kop khun krap
go raibh maith agat
mochchakkeram
terima kasih
grazie
arigatô
takk
dakujem
спасибо
Баярлалаа
faafetai lava
kiitos
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dhanyavad
hvala
maururu
köszönöm
gracie
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barka
wелalin
tack
spas
dank je
vinaka
спасибі
blagodaram
misaotra
matondo
paldies
grazzi
mahalo
tapadh leat
хвала
asante
manana
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didi madloba
najis tuke
kam sah hamnida
rahmat
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