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DE LA RECHERCHE À L'INDUSTRIE
cea

Including neutrinos in standard perturbation theory

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Nonlinear evolution of the large-scale structure of the universe: theory meets expectations

N_{eff} and M_ν are everywhere

- During the radiation-dominated era:

$$\rho_r = \rho_\gamma + \rho_\nu = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\mathrm{eff}}\right] \rho_\gamma .$$

→ Photons AND neutrinos fix the expansion rate at early times.

→ N_{eff} has an impact on the primordial abundances of light elements.

- Neutrinos with a mass lying between 10^{-3} eV and 1 eV are relativistic at matter-radiation equality and non-relativistic today.

→ M_ν has an impact either on z_{eq} or on $\Omega_{m,0}$ (depending on the parameters one decides to keep fixed).

→ In principle, the CMB spectrum is sensitive to M_ν (background effects + effects on secondary anisotropies).

But the CMB alone is not sufficient to constrain sub-eV neutrino masses.

Massive neutrinos and the linear matter power spectrum

Taken from the book *Neutrino Cosmology* (J.Lesgourgues, G. Mangano, G. Miele and S. Pastor, 2013)

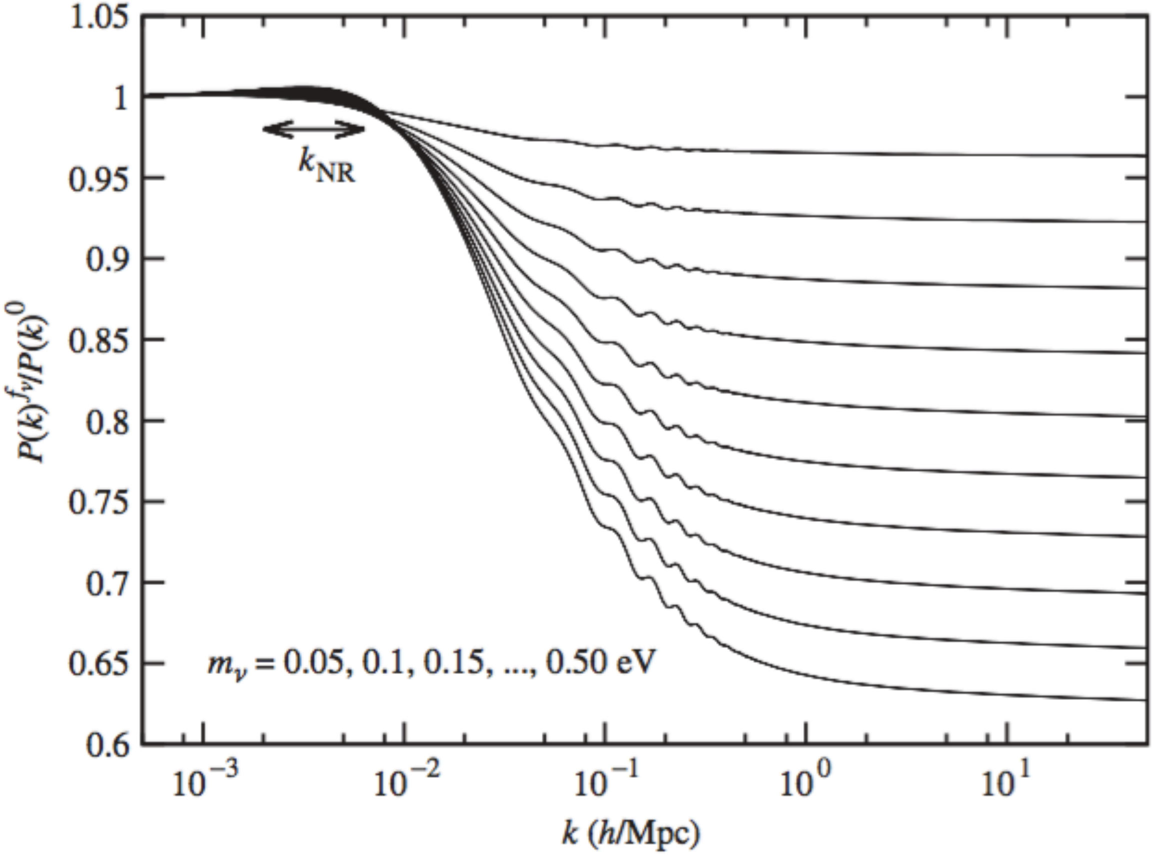
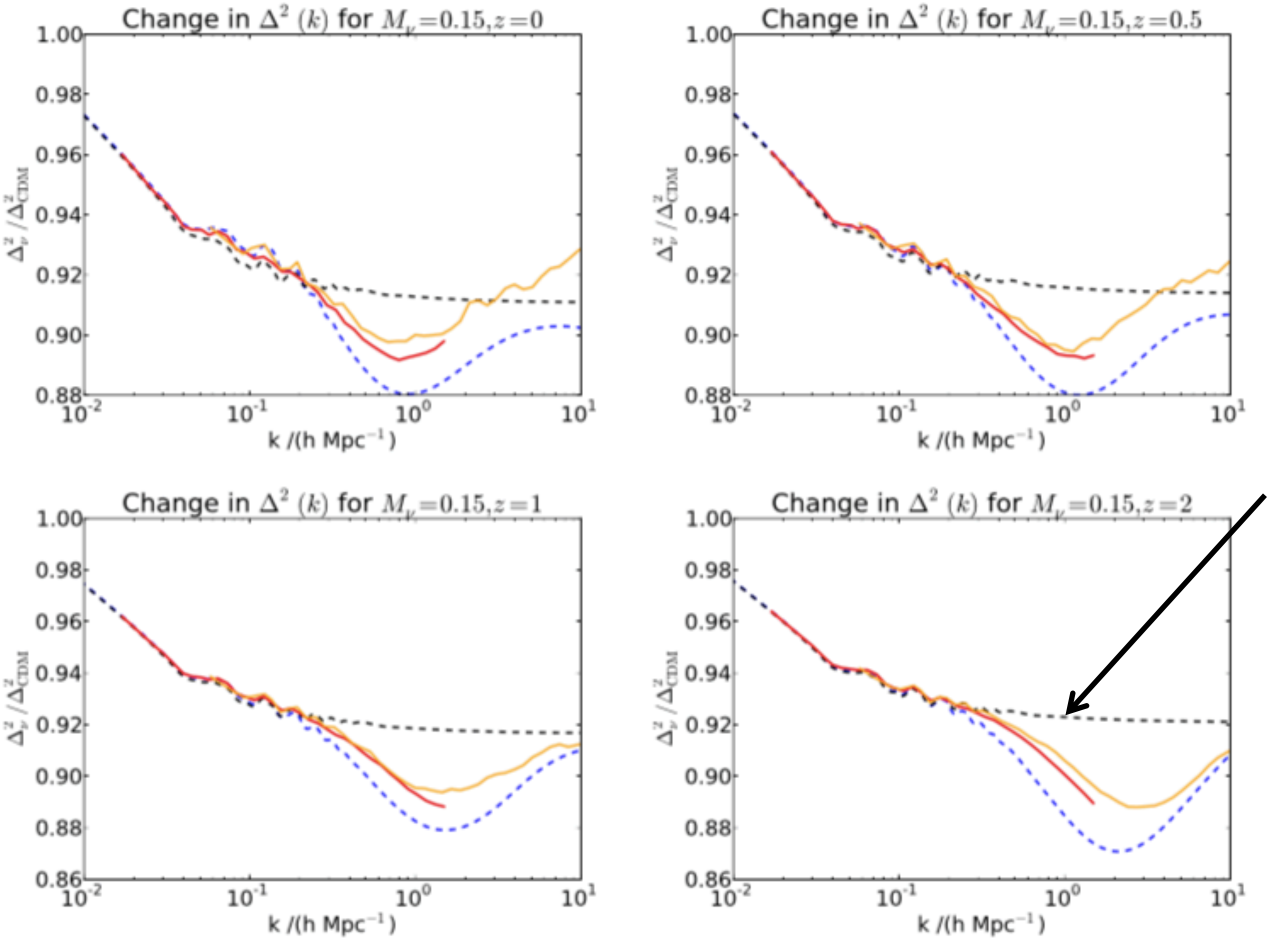


Figure 6.5 Steplike suppression of the matter power spectrum due to neutrino mass. The power spectrum of a ΛCDM model with two massless and one massive species has been divided by that of a massless model, for several values of m_ν between 0.05 eV and 0.50 eV, spaced by 0.05 eV. All spectra have the same primordial power spectrum and the same parameters ($\Omega_{\rm M}$, $\omega_{\rm M}$, $\omega_{\rm B}$).

Massive neutrinos and the non-linear matter power spectrum

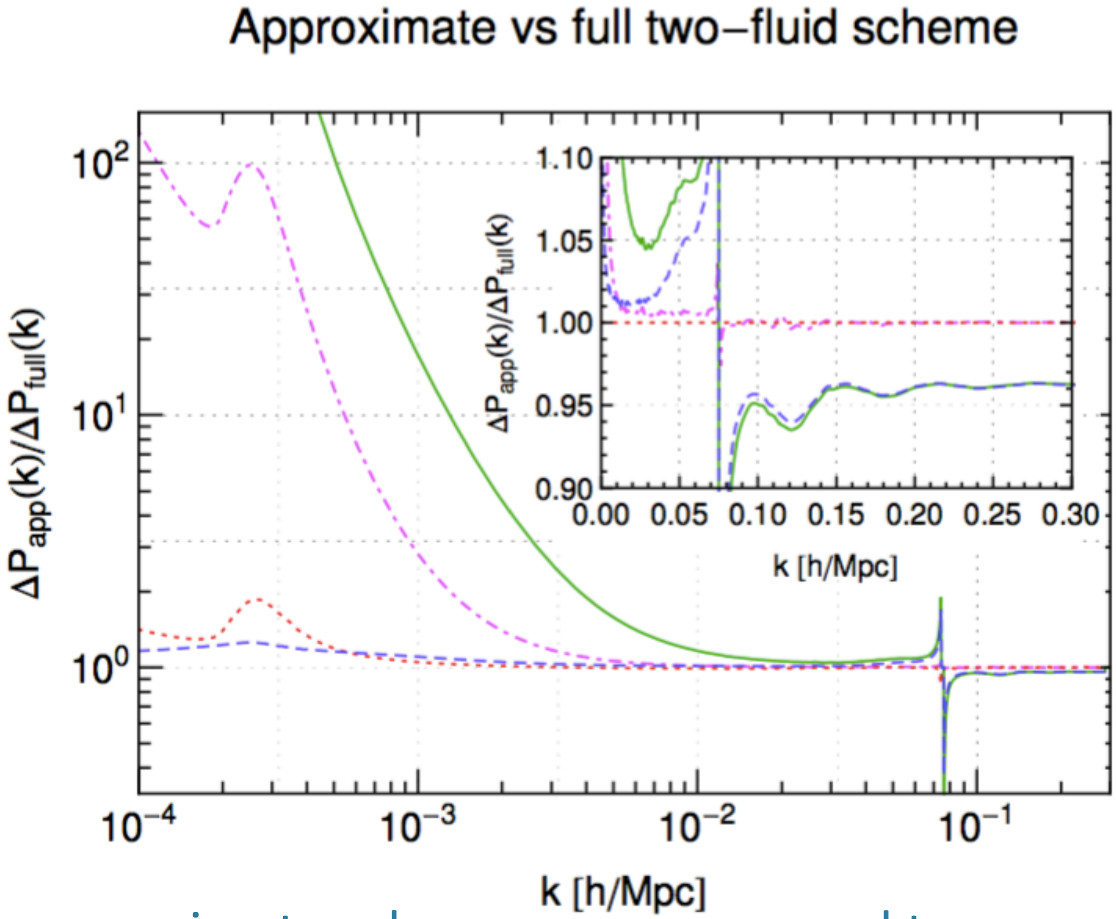


Linear prediction

Numerical simulations showing the effect of massive neutrinos on the non-linear matter power spectrum (*m* = 0.15 eV)

Authors: S. Bird *et al.* (arXiv: 1109.4416)

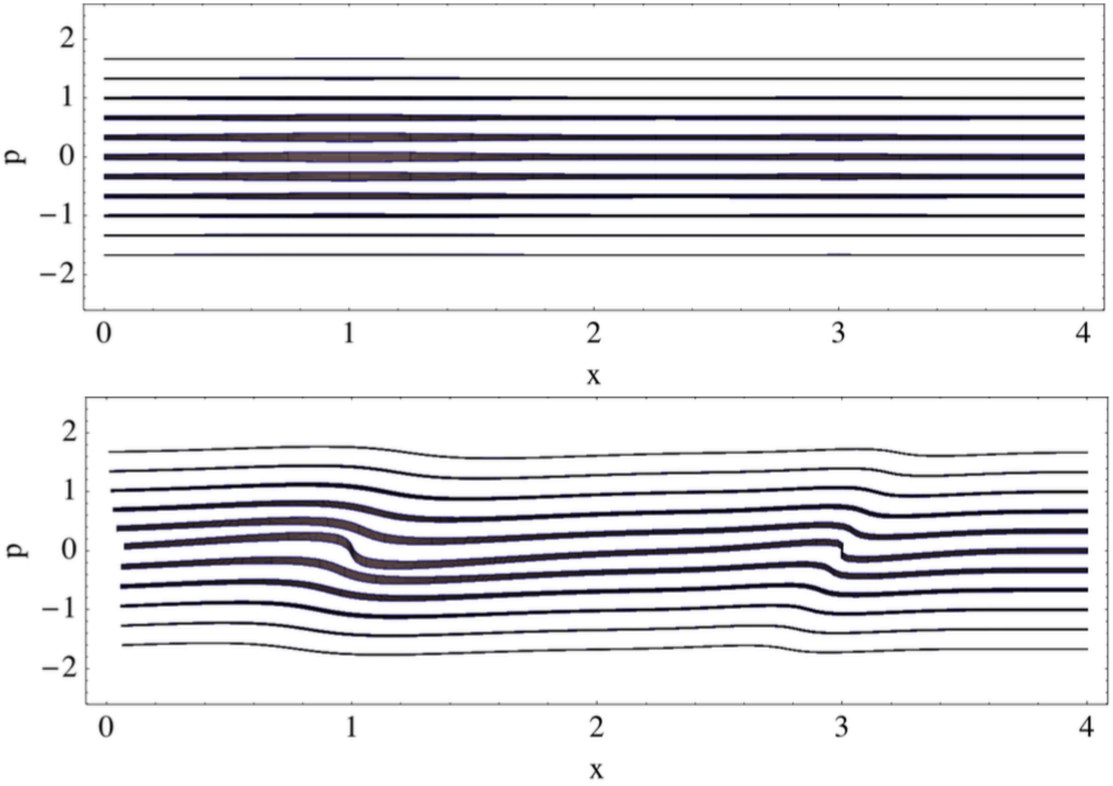
Why should neutrino perturbations be treated non linearly?



Different approximate schemes are compared to a one in which non-linearities of neutrinos are taken into account. The curves represent ratios of non-linear contributions to the matter power spectrum.

Authors: D. Blas *et al.* (arXiv: 1408.2995)

A multi-flow approach to study non-cold species beyond the linear regime



initial time

later time

initial momentum
(label of the flow)

- Total distribution function: $f^{\mathrm{tot}}(\eta, \mathbf{x}, \mathbf{p}) = \sum_{\vec{\tau}} f_{\vec{\tau}}(\eta, \mathbf{x}, \mathbf{p}).$
- One density field per flow: $n_c(\eta, \mathbf{x}; \vec{\tau}) = \int \mathrm{d}^3 p_i \; f_{\vec{\tau}}(\eta, x^i, p_i).$

- In each flow, $P_i(\eta, \mathbf{x}; \vec{\tau}) = \dfrac{\int \mathrm{d}^3 p_i \;\, f_{\vec{\tau}}(\eta, x^i, p_i) p_i}{\int \mathrm{d}^3 p_i \;\, f_{\vec{\tau}}(\eta, x^i, p_i)}\,.$

arbitrary function

- More generally, $\mathcal{F}\left[P_i(\eta, \mathbf{x})\right] n_c(\eta, \mathbf{x}) = \displaystyle\int \mathrm{d}^3 p_i \;\, f(\eta, x^i, p_i) \;\, \overset{\downarrow}{\mathcal{F}} \left[p_i\right].$

- The physical quantities of interest can be expressed in terms of our fields:

$$\underbrace{\int \mathrm{d}^3 p_i \, f^{\mathrm{tot}}(\eta, x^i, p_i) \;\, \mathcal{F}(p_i)}_{\text{Boltzmann approach}} = \underbrace{\int \mathrm{d}^3 \tau_i \, n_c(\eta, \mathbf{x}; \tau_i) \;\, \mathcal{F}(P_i(\eta, \mathbf{x}; \tau_i))}_{\text{our approach}}\,.$$

- **In each flow**, the equation of motion of the density field is

$$\boxed{\frac{\partial}{\partial \eta} n_c + \frac{\partial}{\partial x^i}\left(\frac{P^i}{P^0} n_c\right) = 0},$$

where $P^i = g^{ij} P_j$ and P^0 is defined so that $P^\mu P_\mu = -m^2$.

- **In each flow,** $T^{\mu\nu} = -P^\mu J^\nu.$

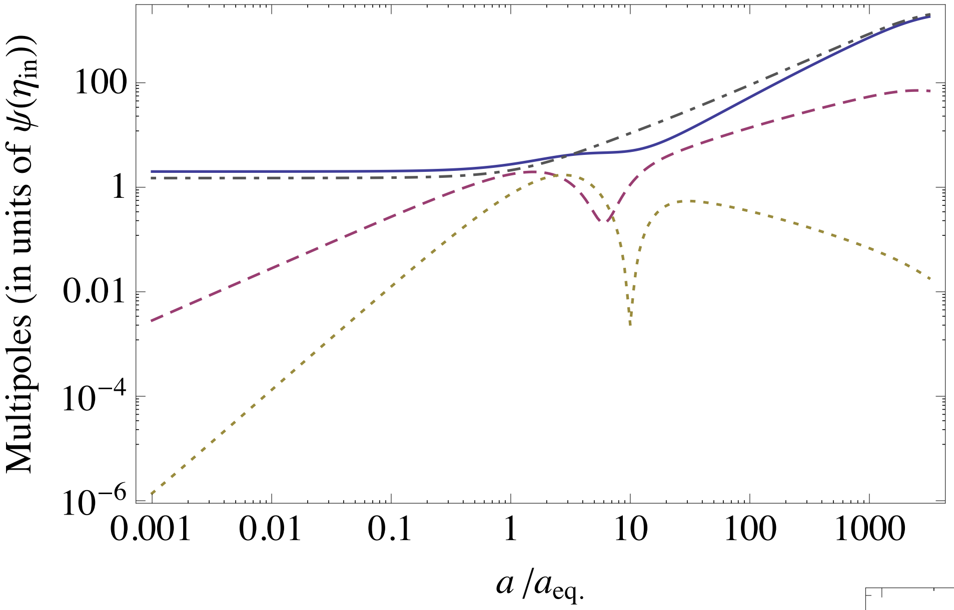
energy-momentum tensor

particle four-current

Combined conservation laws impose

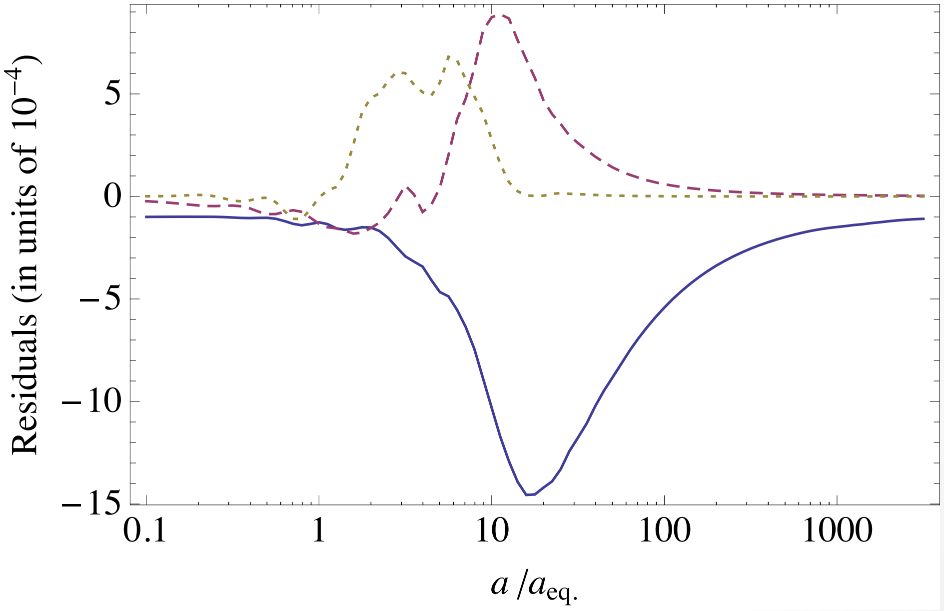
$$\boxed{P^\nu \partial_\nu P_i = \frac{1}{2} P^\sigma P^\nu \partial_i g_{\sigma\nu}}.$$

$$(l_{\max} = 6, N_\mu = 12, N_{\mathrm{q}} = N_\tau = 40, k = k_{\mathrm{eq}} = 0.01h/\mathrm{Mpc}, m = 0.3\ \mathrm{eV}).$$



Multipoles computed using **our approach**.
Solid line: energy density contrast.
Dashed line: velocity divergence.
Dotted line: shear stress

Relative differences (in units of 10^{-4}) in comparison with the Boltzmann approach



More details can be found in
arXiv: 1311.5487 (H. Dupuy and F. Bernardeau).

USEFUL PROPERTIES ON SUBHORIZON SCALES

- In a perturbed Friedmann-Lemaître metric, the equations read

$$\frac{\partial}{\partial\eta}n_c + \frac{\partial}{\partial x^i}\left(\frac{P^i}{P^0}n_c\right) = 0,$$

$$\frac{\partial P_i}{\partial\eta} + \frac{P^j}{P^0}\frac{\partial P_i}{\partial x^j} = a^2(\eta)\left[-P^0\partial_i A + P^j\partial_i B_j + \frac{1}{2}\frac{P^jP^k}{P^0}\partial_i h_{jk}\right].$$

- In the **subhorizon** limit, they become

initial momentum of the flow

$$\mathcal{D}_\eta n_c + \partial_i(V_i n_c) = 0,$$

$$\mathcal{D}_\eta P_i + V_j\partial_j P_i = \tau_0\partial_i A + \tau_j\partial_i B_j - \frac{1}{2}\frac{\tau_j\tau_k}{\tau_0}\partial_i h_{jk},$$

with $\tau_0 = -\sqrt{m^2a^2+\tau_i^2}, \quad \mathcal{D}_\eta = \frac{\partial}{\partial\eta} - \frac{\tau_i}{\tau_0}\frac{\partial}{\partial x^i}$

and $V_i = -\frac{P_i-\tau_i}{\tau_0} + \frac{\tau_i}{\tau_0}\frac{\tau_j(P_j-\tau_j)}{(\tau_0)^2}.$ ← peculiar velocity

GENERALIZATION 1: NO CURL MODES IN THE MOMENTUM FIELD

- On **subhorizon scales** the curl field, defined as

$$\Omega_i = \epsilon_{ijk} \partial_k P_j ,$$

obeys the equation $\mathcal{D}_\eta \Omega_k + V_i \partial_i \Omega_k + \partial_i V_i \Omega_k - \partial_i V_k \Omega_i = 0.$

→ The curl field is only sourced by itself.

→ For adiabatic initial conditions, the **comoving momentum field can be written as a gradient**.

→ As the velocity field of cold dark matter, it is **entirely characterized by its divergence**.

GENERALIZATION 2: ALL MODE COUPLINGS ARE QUADRATIC

- By analogy with cold dark matter, we introduce

$$\theta_{\tau_i}(\eta, x^i) = -\frac{P_{i,i}(\eta, x^i; \tau_i)}{ma\mathcal{H}}, \quad \delta_{\tau_i}(\eta, x^i) = \frac{n_c(\eta, x^i; \tau_i)}{n_c^{(0)}(\tau_i)} - 1.$$

- In Fourier space, it gives

$$\left(a\partial_a - \mathrm{i}\frac{\mu k\tau}{\mathcal{H}\tau_0}\right)\delta_{\vec{\tau}}(\mathbf{k}) - \frac{ma}{\tau_0}\left(1 - \frac{\mu^2\tau^2}{\tau_0^2}\right)\theta_{\vec{\tau}}(\mathbf{k}) = \frac{ma}{\tau_0}\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\alpha_R(\mathbf{k}_1, \mathbf{k}_2; \vec{\tau})\delta_{\vec{\tau}}(\mathbf{k}_1)\theta_{\vec{\tau}}(\mathbf{k}_2),$$

$$\left(1 + a\frac{\partial_a\mathcal{H}}{\mathcal{H}} + a\partial_a - \mathrm{i}\frac{\mu k\tau}{\mathcal{H}\tau_0}\right)\theta_{\vec{\tau}}(\mathbf{k}) + \frac{k^2}{ma\mathcal{H}^2}\mathcal{S}_{\vec{\tau}}(\mathbf{k}) = \frac{ma}{\tau_0}\int \mathrm{d}^3\mathbf{k}_1\mathrm{d}^3\mathbf{k}_2\beta_R(\mathbf{k}_1, \mathbf{k}_2; \vec{\tau})\theta_{\vec{\tau}}(\mathbf{k}_1)\theta_{\vec{\tau}}(\mathbf{k}_2).$$

- Considering *N* flows, it is useful to introduce the 2*N*-uplet

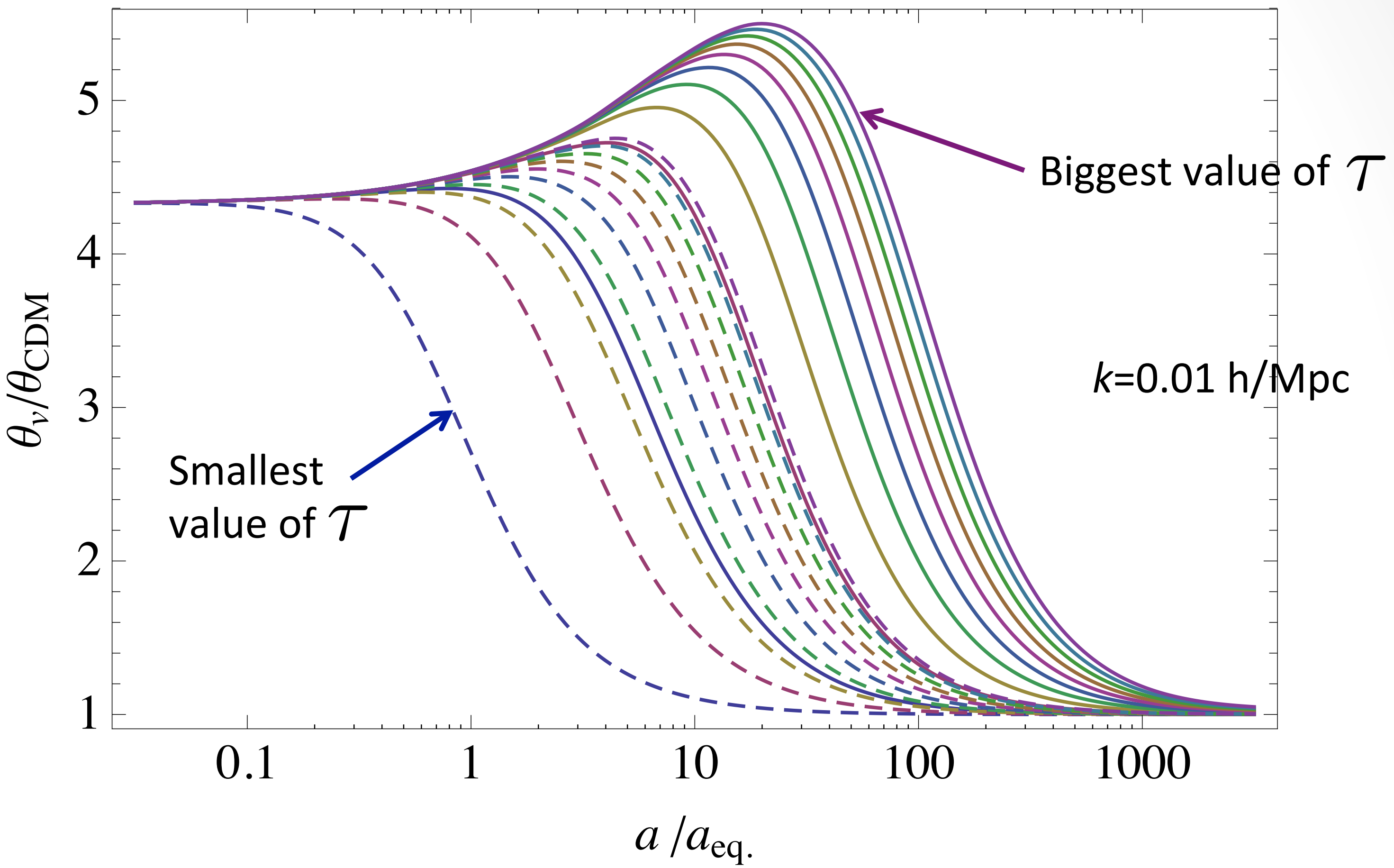
$$\Psi_a(\mathbf{k}) = (\delta_{\tau_1}(\mathbf{k}), \theta_{\tau_1}(\mathbf{k}), \ldots, \delta_{\tau_n}(\mathbf{k}), \theta_{\tau_n}(\mathbf{k}))^T.$$

- The resulting equations is

$$\boxed{a\frac{\partial \Psi_a}{\partial a}(\mathbf{k},\eta) + \Omega_a{}^b(\mathbf{k},\eta)\,\Psi_b(\mathbf{k},\eta) = \gamma_a{}^{bc}(\mathbf{k}_1,\mathbf{k}_2,\eta)\Psi_b(\mathbf{k}_1,\eta)\Psi_c(\mathbf{k}_2,\eta).}$$

The relativistic equation of motion is **formally the same as the equation describing cold dark matter**.

- This study is presented in arXiv: 1411.0428 (H. Dupuy and F. Bernardeau).



Time evolution of the velocity divergence for different values of τ, with $\mu = 0 (0.45 k_{\rm B} T_0 < \tau < 9 k_{\rm B} T_0)$.

Solid lines: $m = 0.05eV$.
Dashed lines: $m = 0.3eV$.

Prospects

- Solving the non-linear system of equations in the eikonal approximation (arXiv:1109.3400, F. Bernardeau et al.).

- Solving the non-linear system of equations in the general case. What is the most suitable technique? Can the time-renormalization group method (arXiv:0806.0971, M. Pietroni) be extended to the study of a collection of flows?

- Using our results to visualize the gravitational collapse of neutrinos for given initial distributions of matter. Can we learn something about the space distribution of voids?