High precision simulations for high precision cosmology:
a discussion about (un?)controlled numerical experiments

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Plan

- Why do we use cosmological simulations ?
- What is the physical problem to treat?
- Dark matter simulations techniques (true simulations, I mean)
- Limits of N-body simulations
- Perspectives (advertisement)

Why do we use (N-body) simulations ?

To solve the dynamics of a physical system for which there is no analytical solution

In practice:

- To understand the dynamics of a vey complex system, e.g. a dark matter halo (hence NFW) or a galaxy

- To check the validity of some approximation, e.g. cosmological perturbation theory at large scales (BAOs) or some phenomenological model, e.g. the halo model

- To generate realistic mock observations, e.g. a mock galaxy catalogue

Example: old plot of the power spectrum of the large scale galaxy distribution

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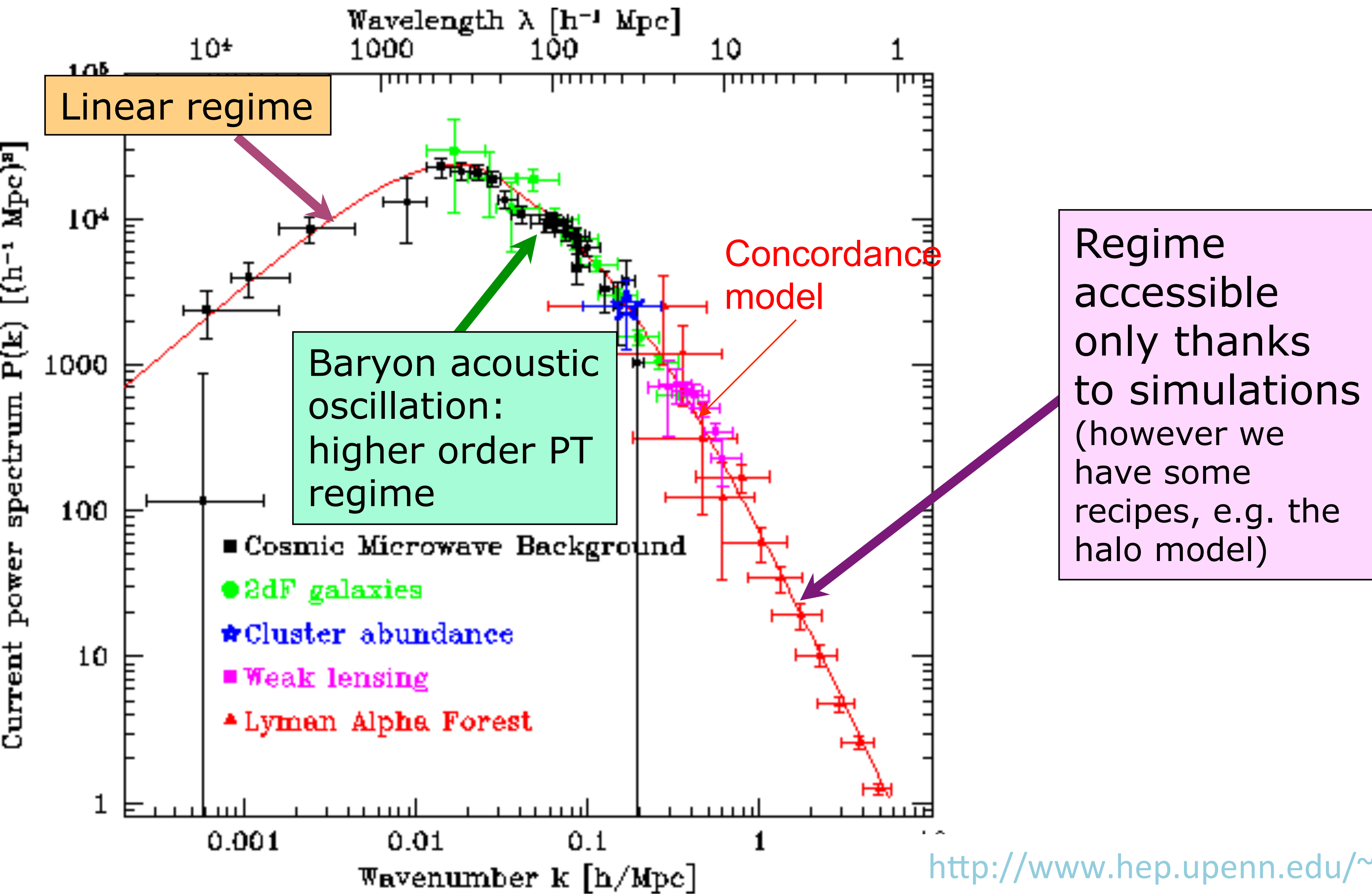


http://www.hep.upenn.edu/~max

What is the physical problem to treat?

Dark matter : Vlasov

$$\frac{\partial f}{\partial t} + \mathbf{u}.\nabla f - \nabla\Phi.\nabla_u f = 0$$

Gas : Euler equations

$$\frac{\partial \rho_\mathrm{b}}{\partial t} + \nabla.(\rho_\mathrm{b}\mathbf{u}) = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla).\mathbf{u} = -\nabla\Phi - \frac{\nabla p}{\rho_\mathrm{b}}$$

$$\frac{\partial \varepsilon}{\partial t} + \mathbf{u}.\nabla\varepsilon = -\frac{p}{\rho_\mathrm{b}}\nabla.\mathbf{u}$$

Equation of state

$$p = (\gamma - 1)\varepsilon\rho_\mathrm{b}$$

Poisson equation

$$\nabla^2\Phi = 4\pi G\left[\int f d^3u + \rho_\mathrm{b}\right]$$

+ the ugly physics I wont discuss much except to feel sorry for myself with e.g. my beautiful 10th order PT results: gas heating/cooling processes, star formation, feedback, etc

Note on how to take into account of the expansion of the Universe in the previous equations

``Supercomoving'' coordinates:

The equations remain nearly unchanged

(Martel & Shapiro 1998, MNRAS 297, 467)

$$
\begin{aligned}
d\tilde{t} &= H_0 \frac{dt}{a^2} \\
\tilde{x} &= \frac{1}{a}\frac{x}{L} \\
\tilde{\rho} &= a^3 \frac{\rho}{\Omega_0 \rho_c} \\
\tilde{p} &= a^5 \frac{p}{\Omega_0 \rho_c H_0^2 L^2} \\
\tilde{\mathbf{u}} &= a \frac{\mathbf{u} - H\mathbf{x}}{H_0 L}
\end{aligned}
$$

$$\tilde{\nabla}^2 \tilde{\Phi} = \frac{3}{2} a \Omega_0 (\tilde{\rho} - 1)$$

3 approaches to simulate the Universe

1) try to do everything including the treatment of the ugly physics of the baryons in the simulations: *given the complexity of all the processes at game, it is not yet possible to simulate a large volume of the Universe*

2) forget about the baryons and simulate only dark matter which follows ``only'' Vlasov-Poisson equations: *handy to test perturbation theory predictions, possible to generate very big samples*

3) Feel guilty about 2) and try to ``paint'' dark matter simulations by taking into account in the best way possible the ugly physics, e.g.
   a) semi-analytical models
   b) probabilistic approach using cross-correlations between hydro simulations and dark matter simulations.

Dark matter simulations techniques

Bertschinger, 1998, ARA&A 36, 599
Colombi, 2001 NewAR 45, 373
Dolag et al., 2008, Space Science Review 134, 229

Collisionless dark matter or stars in a galaxy: incompressible fluid in phase-space (**x**,**u**). Direct modeling in phase-space: 6 dimensions !

→ Dark matter: modelled with (macro-)particles which form an Hamiltonian system

$$\frac{d\mathbf{x}_{\mathrm{p}}}{dt} = \mathbf{u}_{\mathrm{p}}$$
$$\frac{d\mathbf{u}_{\mathrm{p}}}{dt} = -\nabla\Phi$$

The modeling in terms of macro-particles induces *N*-body relaxation effects due to particle-particle collisions

→ A softening parameter ε is needed at small scales: each dark matter particle is a ``cloud'' of typical (possibly varying) size ε or, equivalently, the force is softened at scales smaller or of the order of ε

Various types of codes

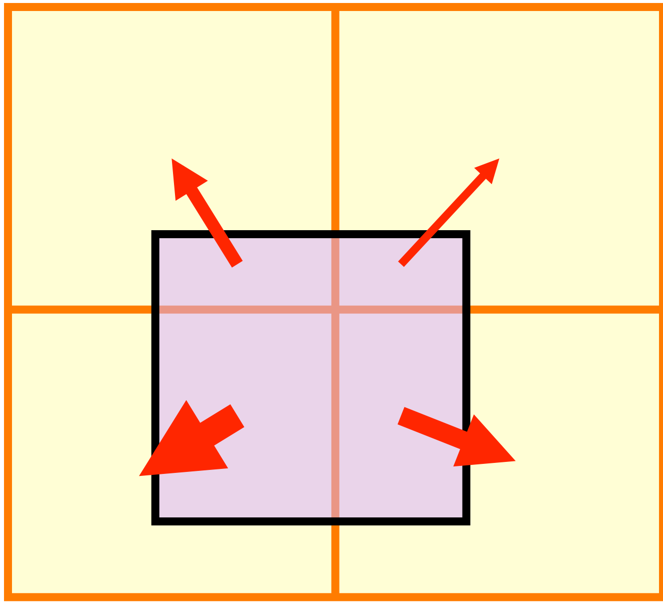
All the codes basically differ by the way Poisson equation is solved

1) **Brute force** with all the interactions between particles calculated (PP)

2) **PM code :** the ``plasma physicist approach''

- density is calculated on a grid of fixed resolution by projecting the particles with some interpolation procedure e.g. CIC or higher order (TSC)
- Poisson equation is simply solved with FFT
- Force reinterpolated on each particle with dual interpolation
- Softening is therefore roughly given by cell size.

CIC (cloud in cell) scheme

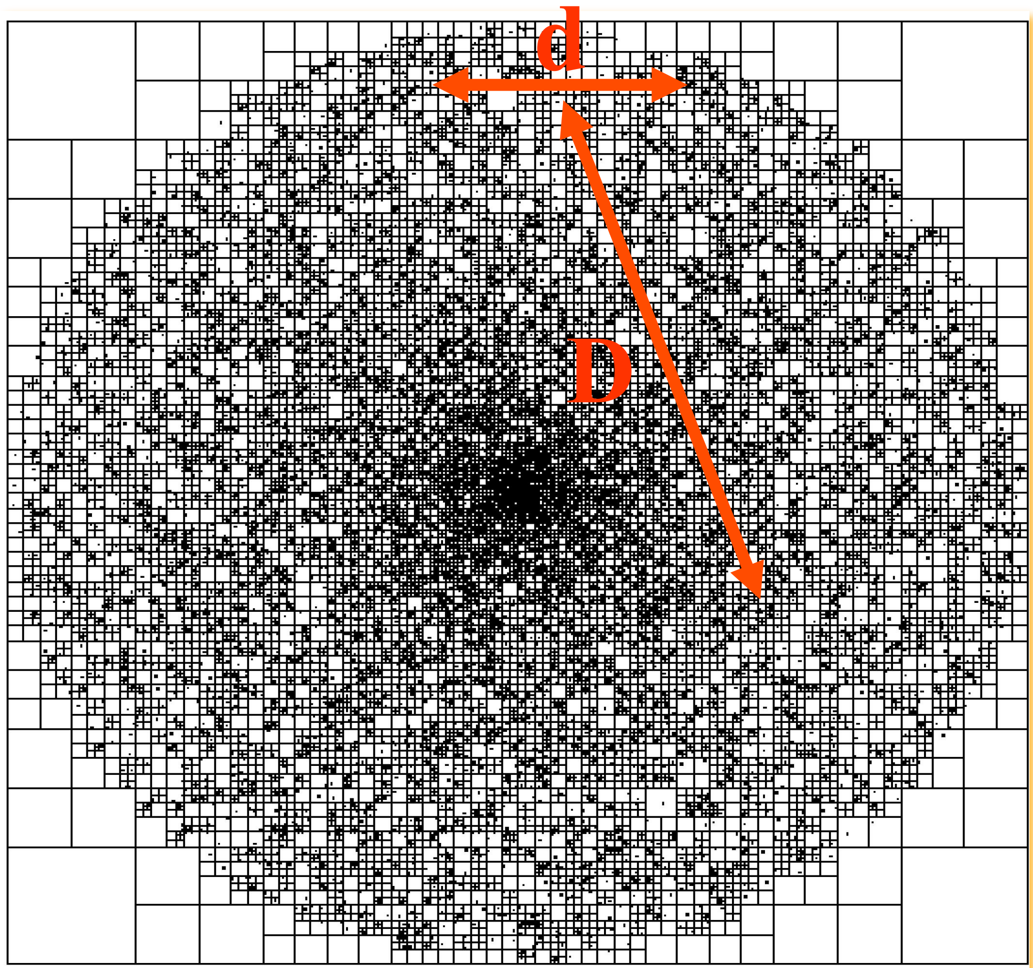


3) Treecode: what is far can be summarized

Appel 1985, SIAM 6, 85; Barnes & Hutt, 1986, Nature 324, 446
GADGET1 : Springel et al. 2001, NewA 6, 79

Classical implementation:

- Hierarchical division of space on an ``oct tree'', until there is only one or zero particle per box.
- A box=a macro-particle if $d/D < \theta$, otherwise the box is divided in 8 subboxes and so on.



A``quad tree''
Salmon & Warren 1994, Int. J. Supercomputer Applic. 8, 129

3) Hybrid methods:

- **P^3M :** PP + PM. The PM force is supplemented with a small scale contribution by direct local summation

  Hockney & Eastwood, 1981; Efstathiou et al. 1985, ApJS 57, 241

- **treePM :** same as P3M but with a local treecode to augment the resolution: faster ?

  Bagla, 2002, JApA 23, 185 GADGET2: Springel 2005, MNRAS 364, 1105

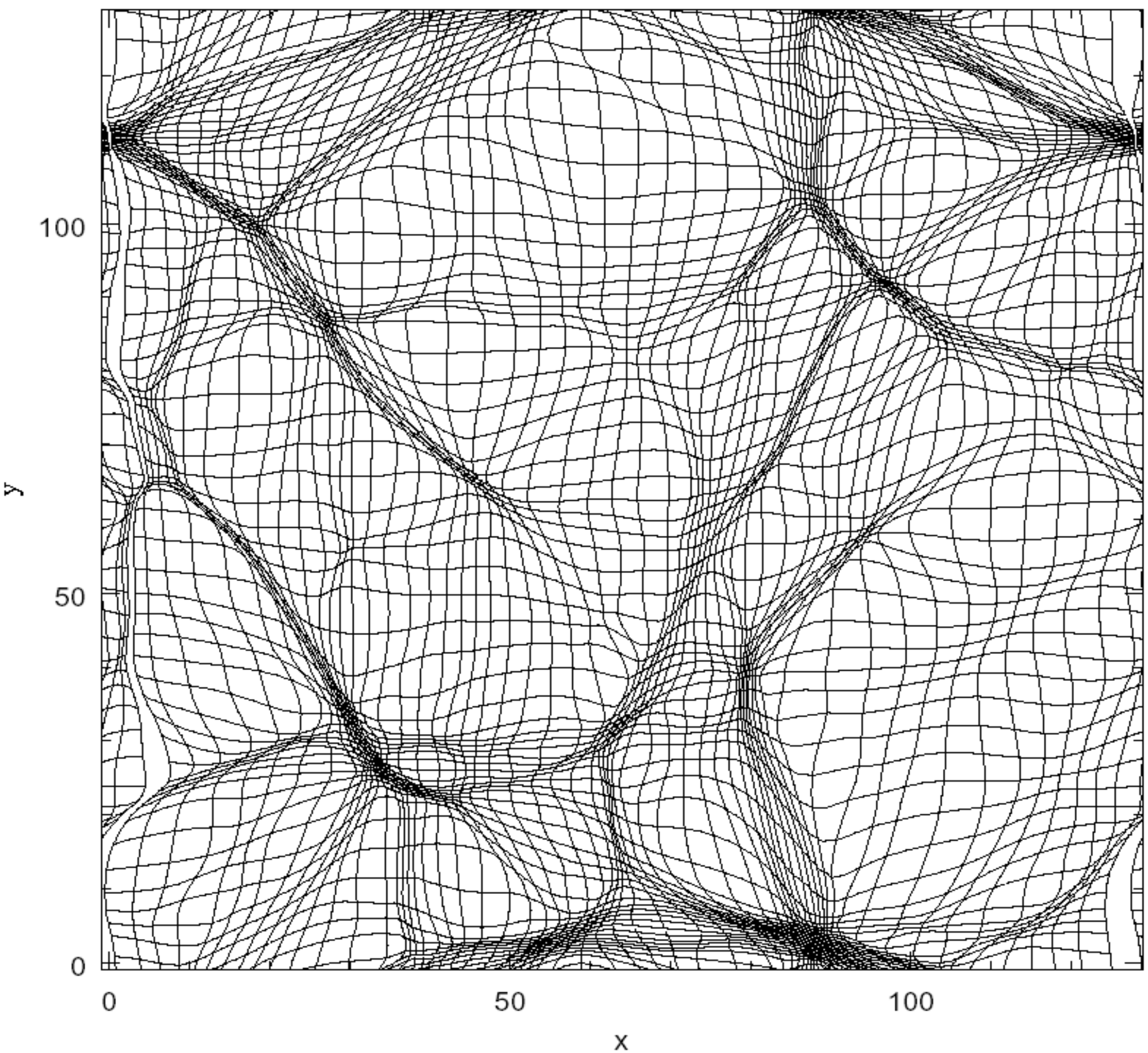
- **AMR :** local refinement of the PM grid. 2 methods: "patch method" (hierarchy of embedded rectangular grids) or ART (adaptive refinement tree). Note : AP^3M : P^3M code with grid refinement.

  Kravtsov et al. 1997, ApJS 111, 73 RAMSES: Teyssier 2002, A&A 385, 337

- **Lagrangian approach** : the PM grid changes shape according to the flow

  Gnedin, 1995, ApJS 97, 231; Pen, 1995, ApJS 100, 269

``Lagrangian'' approach

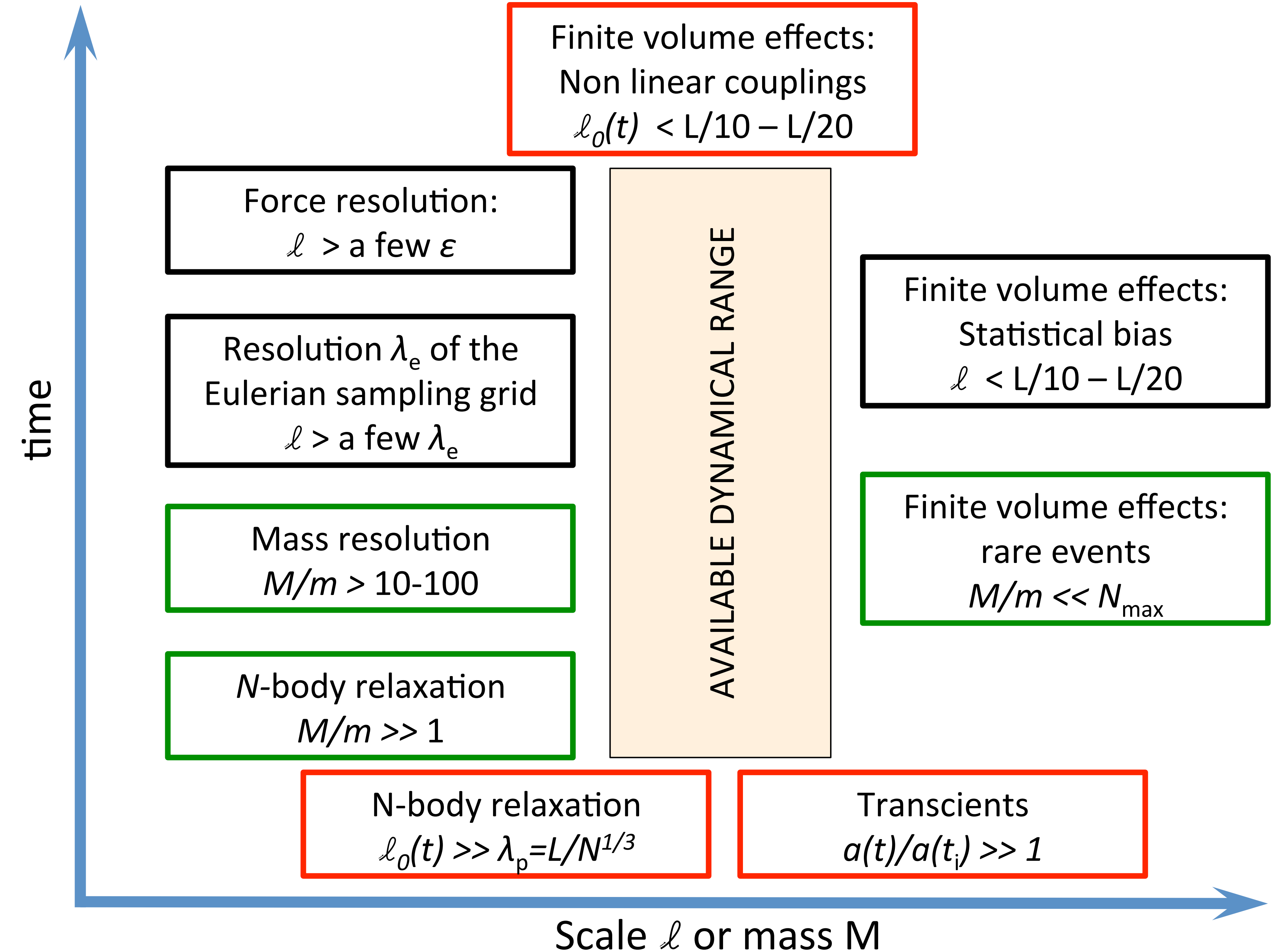


Pen 1995

Limits of N-body simulations

Possibly annoying features in each techniques

- **Brute force :** well forget about it, way too costly
- **PM :** cheap, can provide robust results if used wisely but low spatial resolution. Probably the best tool to test PT predictions if not trying to model biasing
- **Treecode :** problem with force error calculations: impossible to start simulations at very high redshift
- **AMR :** problem with varying softening: symplecticity is broken: can this be an issue, sometimes? Need more particles than treecode to have same effective mass resolution (in terms of DM halos mass function)
- **P^3M :** small forces errors at the transition between PP and PM regime
- **treePM :** same issue as treecode, but certainly to a much lesser extent, same issue as P^3M



time
Scale ℓ or mass M
Finite volume effects:
Non linear couplings
$\ell_0(t) < L/10 – L/20$
Force resolution:
$\ell >$ a few ε
Resolution λ_e of the
Eulerian sampling grid
$\ell >$ a few λ_e
Mass resolution
$M/m > 10\text{-}100$
N-body relaxation
$M/m >> 1$
AVAILABLE DYNAMICAL RANGE
Finite volume effects:
Statistical bias
$\ell < L/10 – L/20$
Finite volume effects:
rare events
$M/m << N_{max}$
N-body relaxation
$\ell_0(t) >> \lambda_p=L/N^{1/3}$
Transcients
$a(t)/a(t_i) >> 1$

Particularly annoying issues:

Anisotropic growth of fluctuations

- Discreteness effects: memory of the initial particle pattern : see e.g. Marcos et al. 2006, PRD 73, 103507; Joyce & Marcos 2007 PRD 10, 103505 for the grid

=> The correlation length must be sufficiently large compared to the mean interparticle distance to allow for sufficient mixing

- Finite box size effects: wrong mode coupling at large scales: see e.g. Seto 1999, ApJ 523, 24; Takahashi et al., 2008, MNRAS 389, 1675; Nishimichi et al. 2009, PASJ 61, 321

=> Need the box size to be (very) large compared to the correlation length

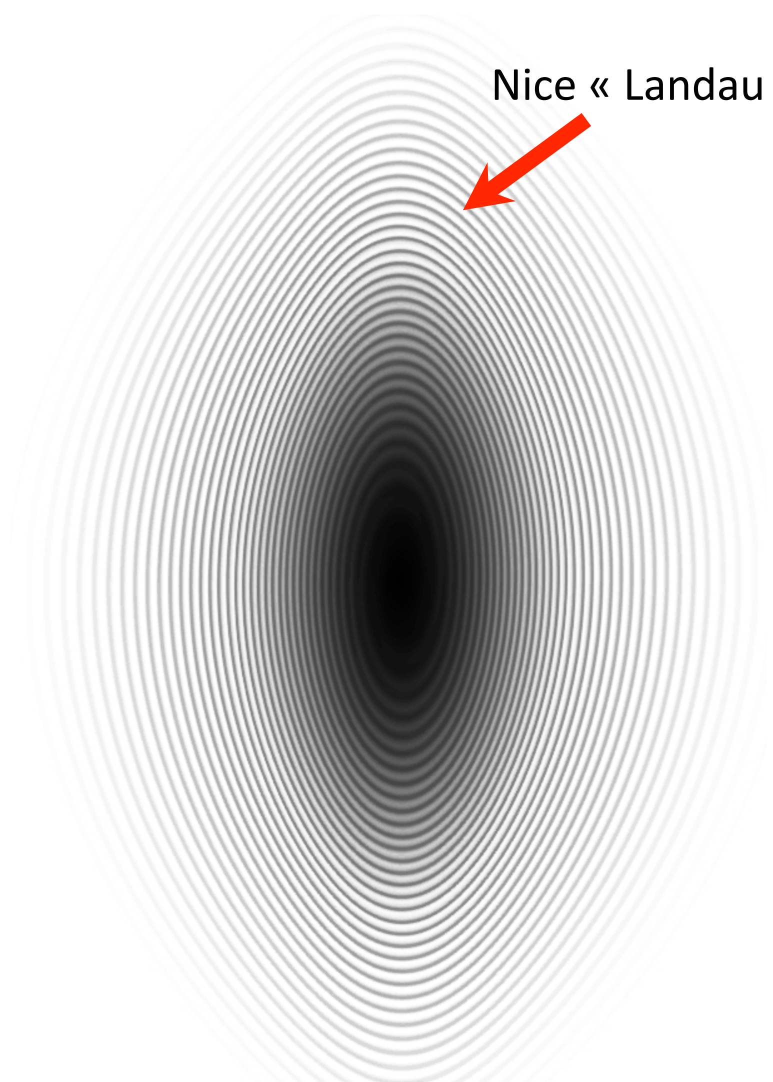
Other transcients due to to Zel'dovich approximation

see e.g. Crocce et al. 2006, MNRAS 373, 369

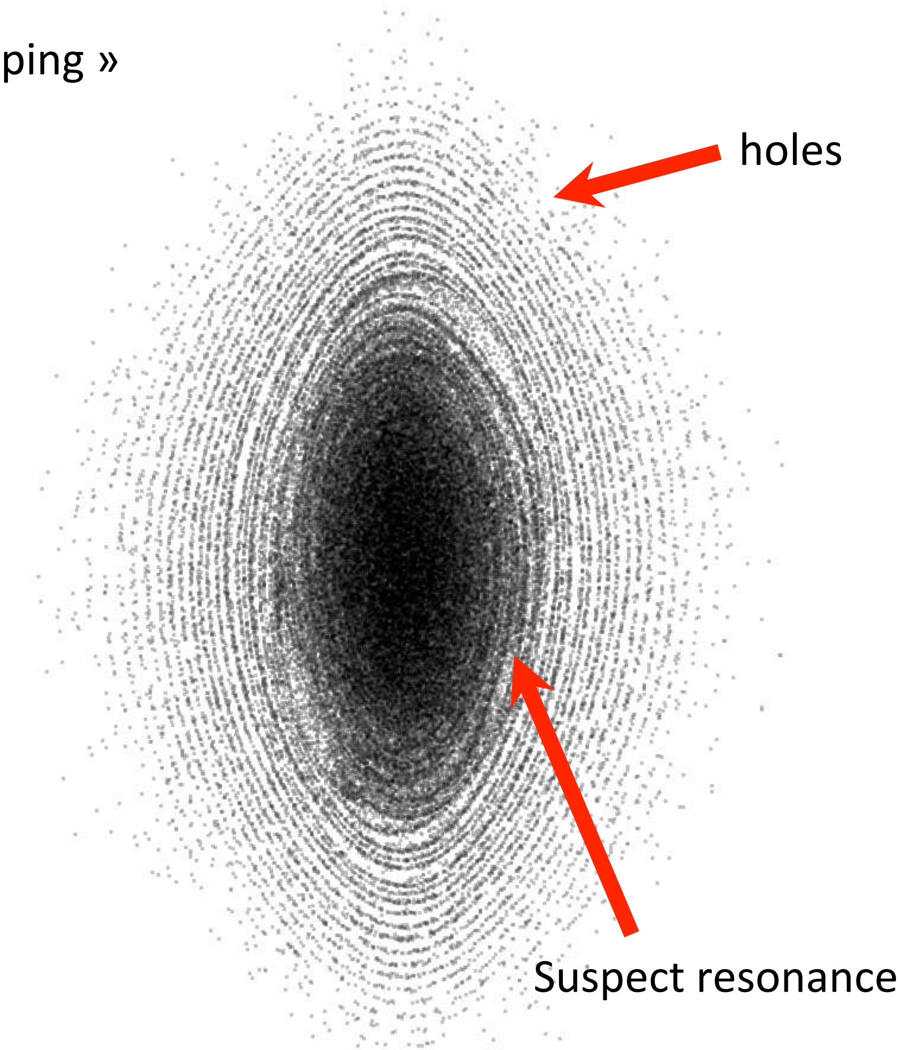
=> must start at very high z or initial conditions must be generated at higher order (2nd order)

N-body simulations are noisy

Example: phase-space of a 1D simulation with Gaussian initial conditions



“Exact” solution with the Waterbag method
Colombi & Touma (2008, 2014)

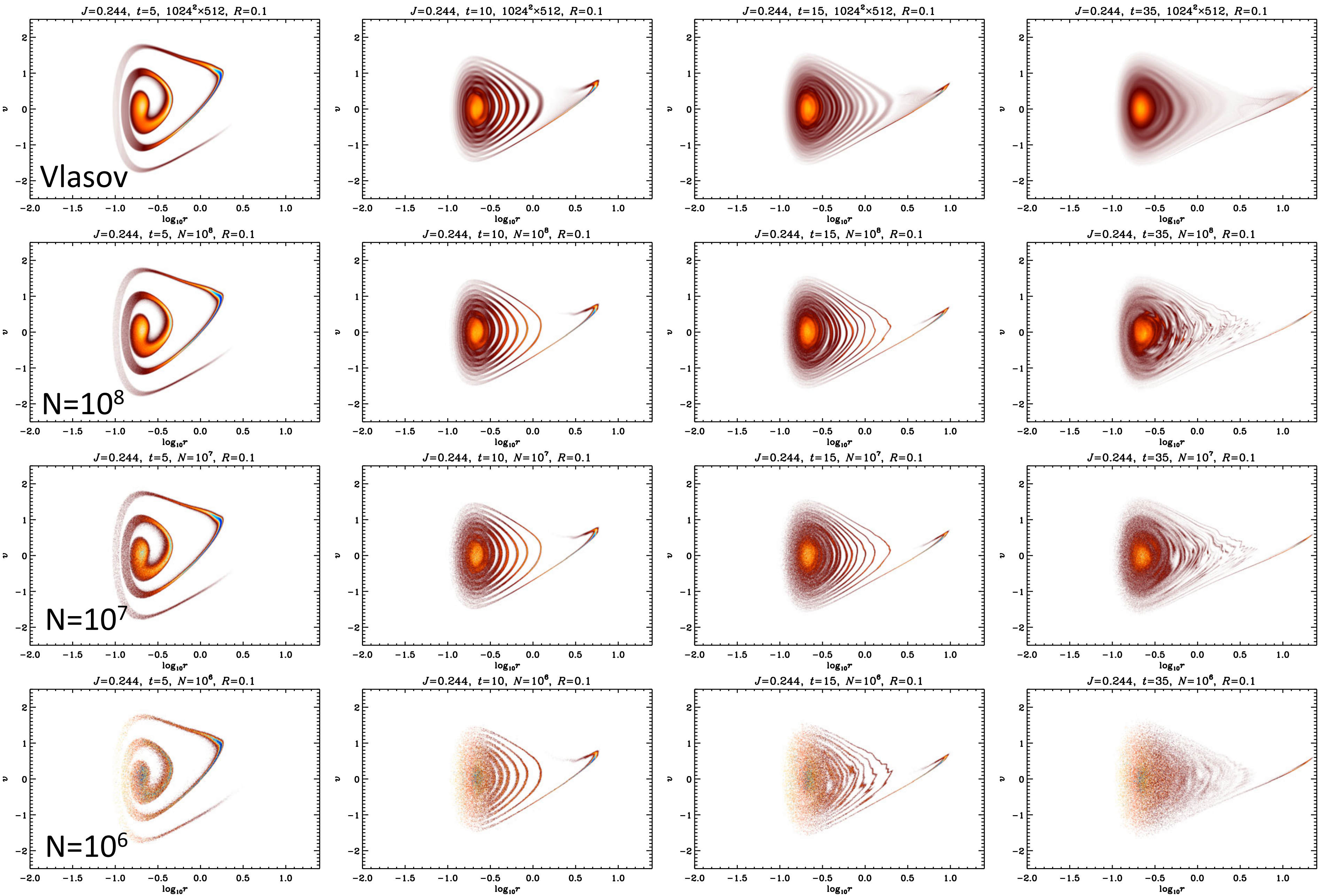


N-body

Spherical collapse of a Hénon sphere: Vlasov code versus N-body: collective shot noise effects

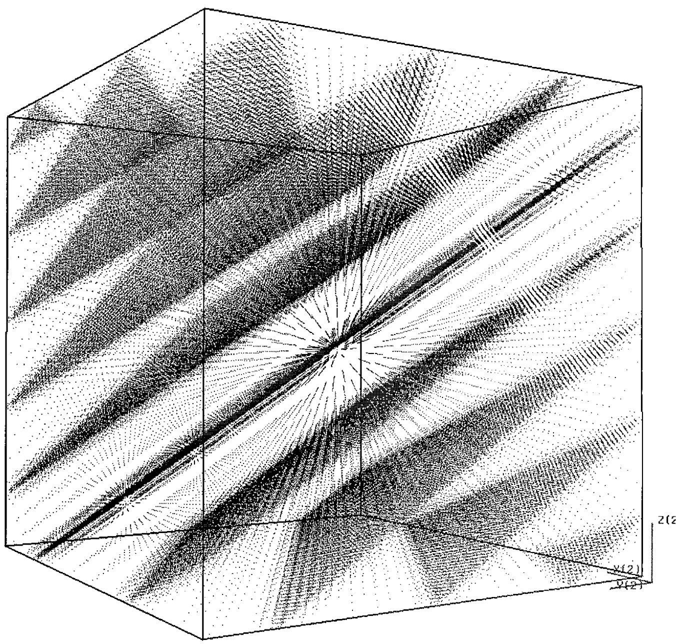
Phase space for a slice of fixed angular momentum

Colombi et al. 2015, MNRAS 450, 3724

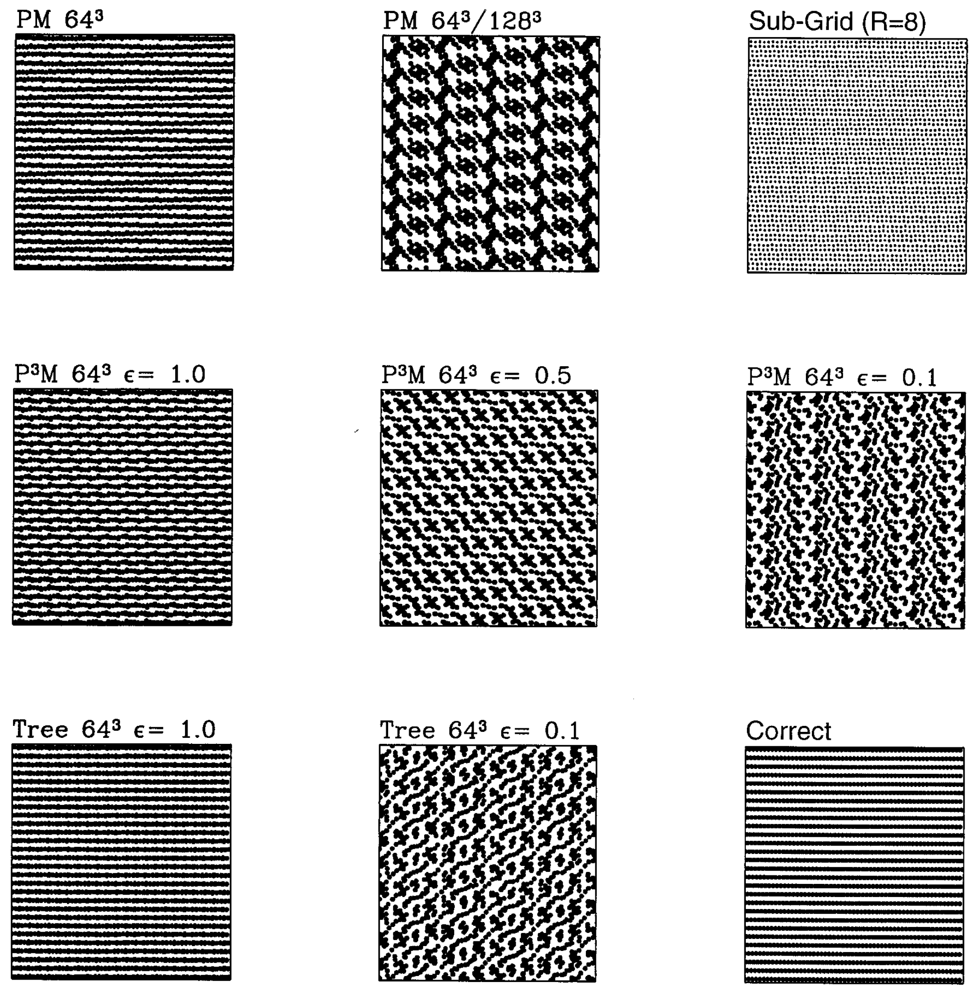


Particularly catastrophic illustration showing that in principle softening length should be larger than interparticle distance: single oblique sine wave

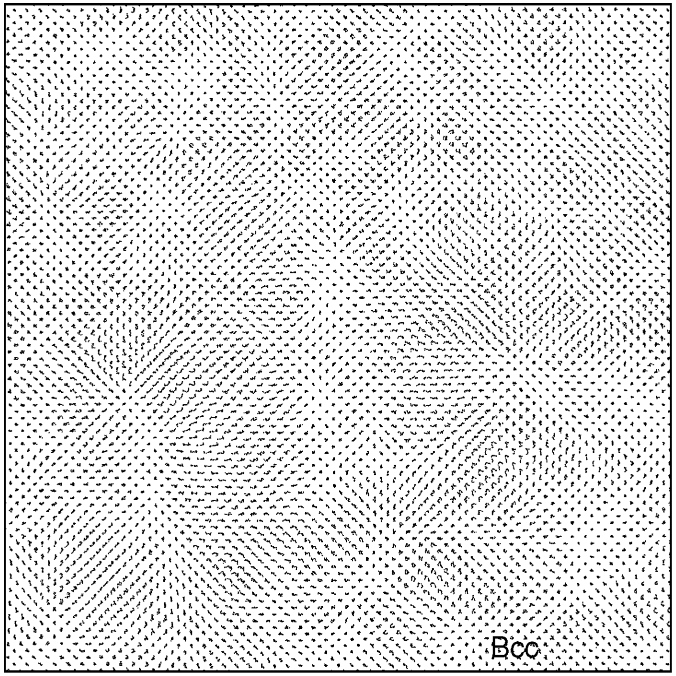
Melott, Shandarin, Splinter & Suto 1997, ApJ 479, L79
Melott, 2007 arXiv 0709.0745

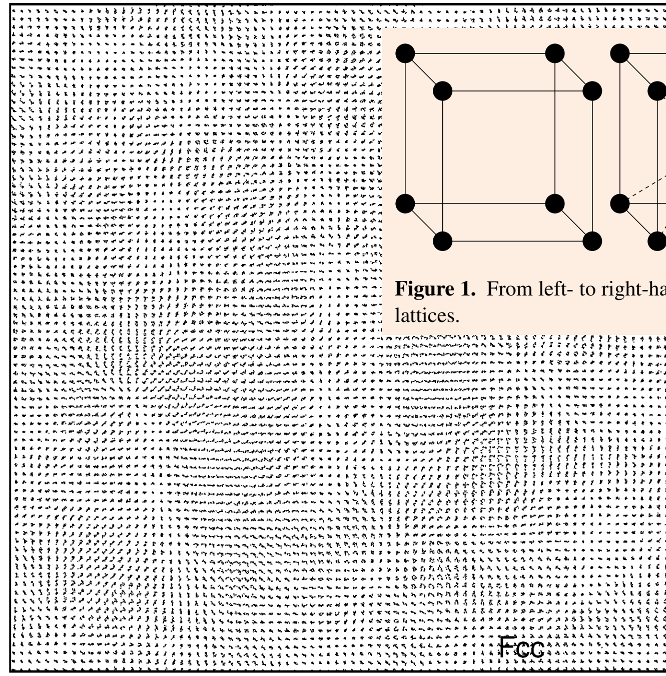


``Correct'' result at the end of the simulation



Various experiments





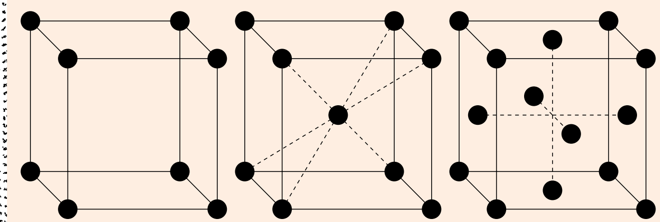
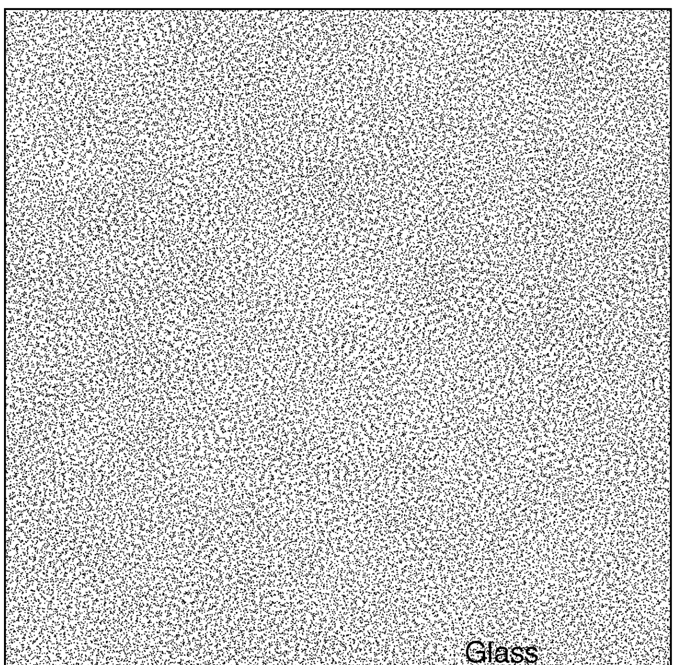
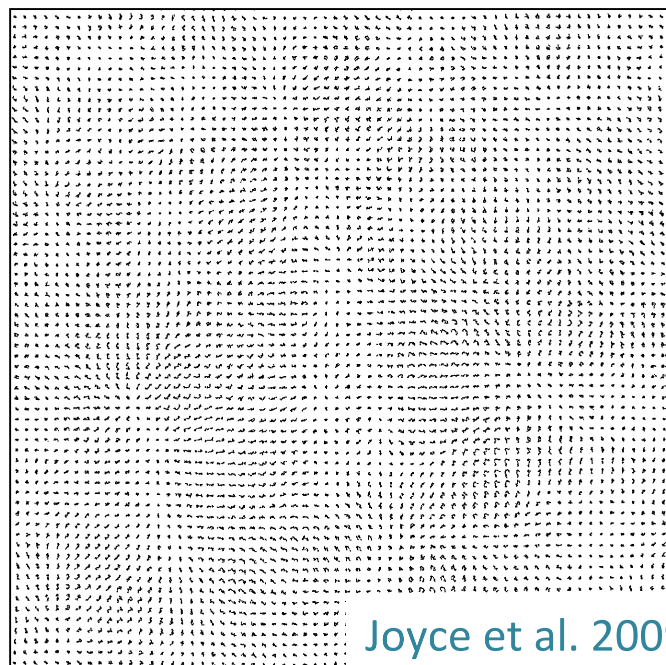
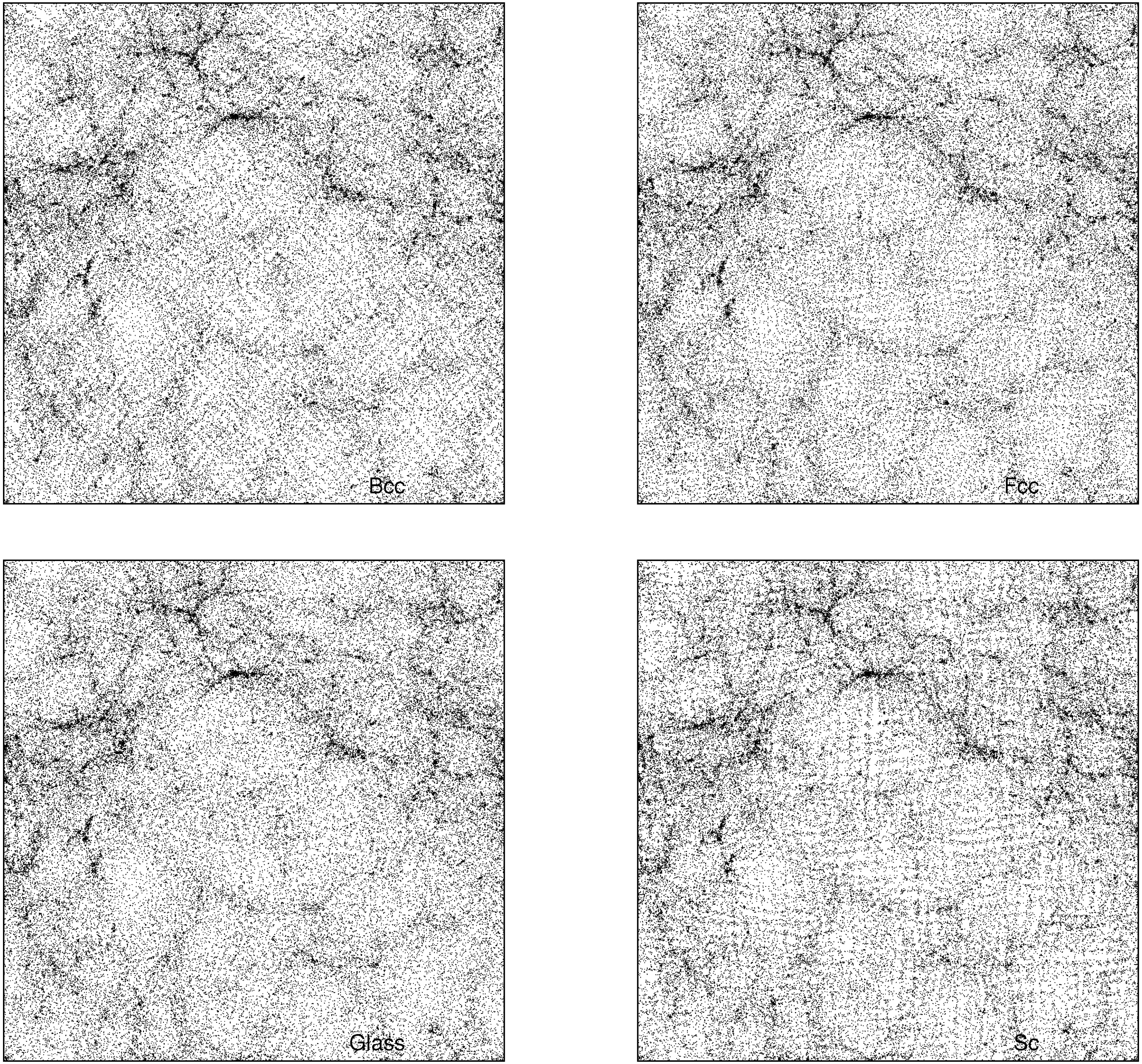


Figure 1. From left- to right-hand side: unit cell of the SC, BCC and FCC lattices.





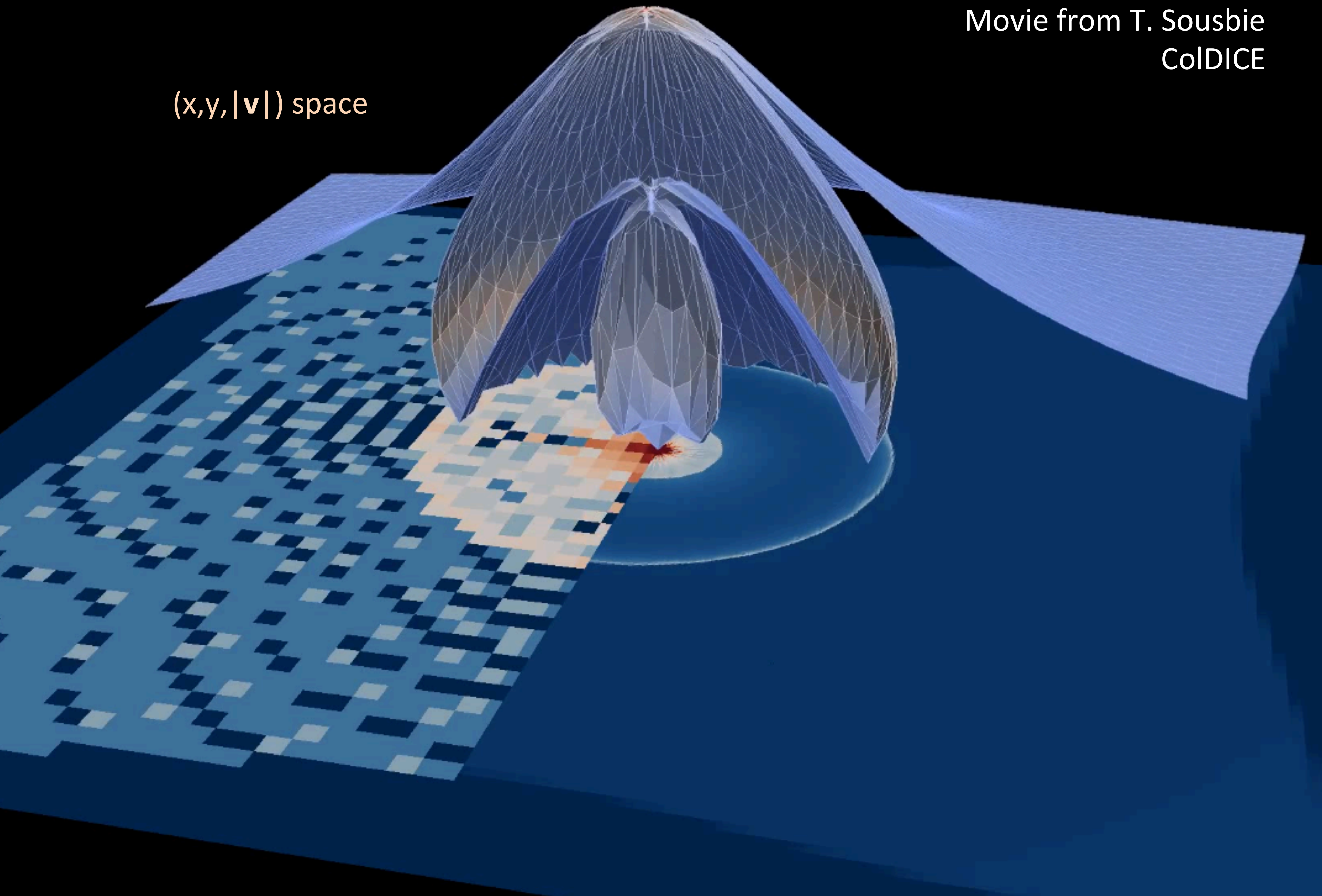
Joyce et al. 2009, MNRAS 394, 751


Bcc
Fcc
Glass
Sc

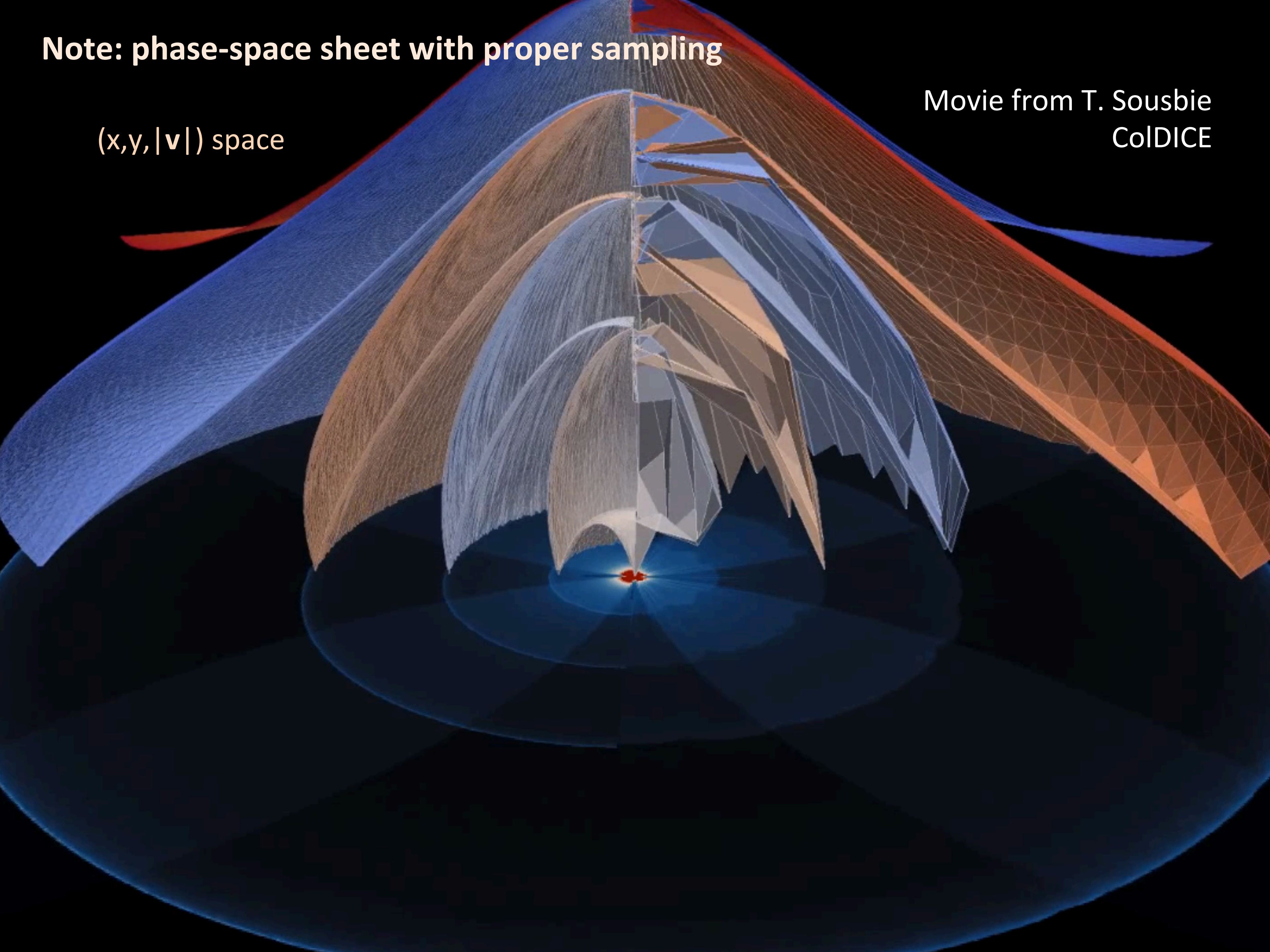
Example in 2D: evolution of the phase-space sheet in a Plummer potential

Movie from T. Sousbie
ColDICE

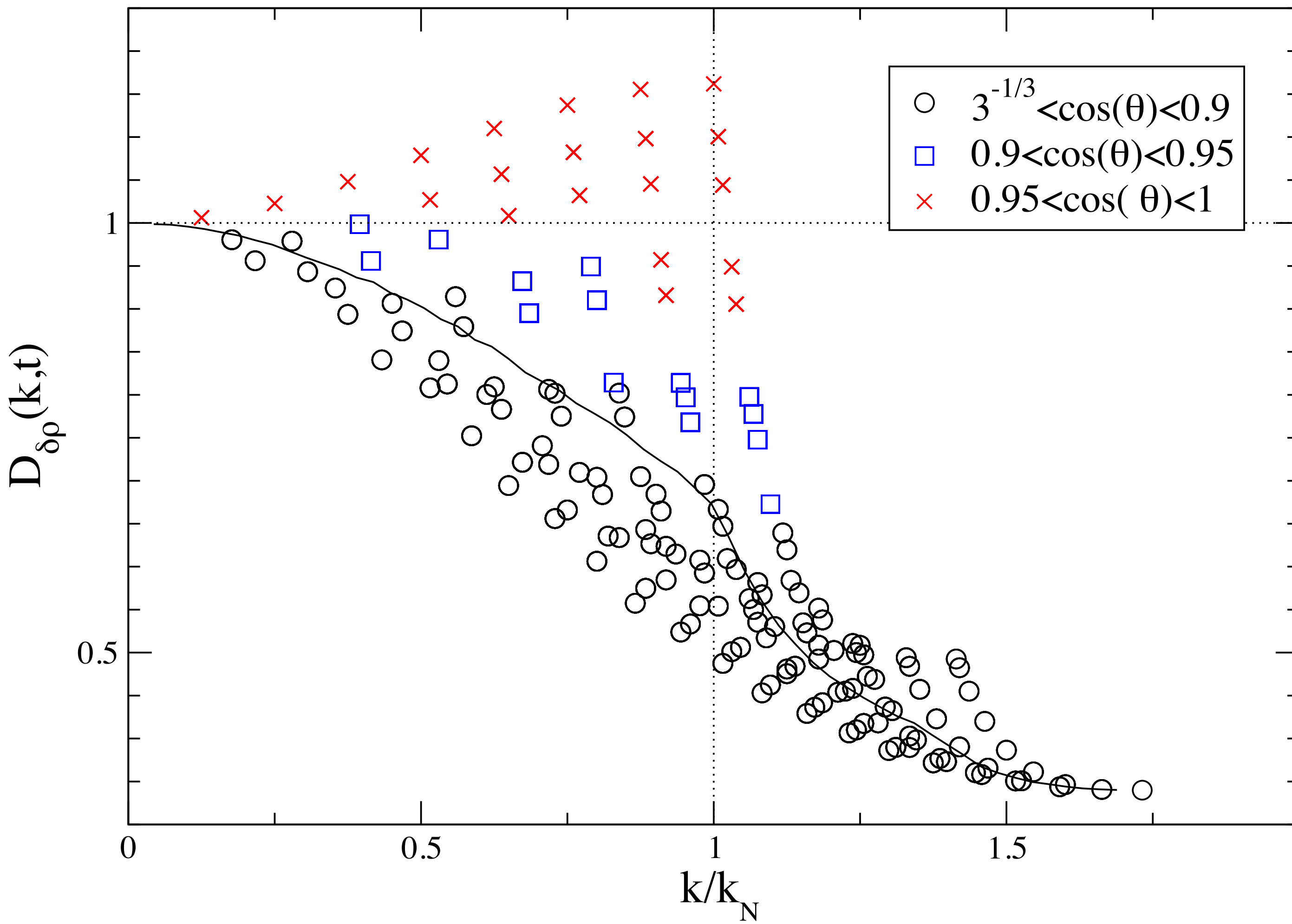
(x,y,|**v**|) space



Note: phase-space sheet with proper sampling



Anisotropic growth of the power-spectrum at small scales



Effects of anisotropy due to a lattice on the power-spectrum

Joyce & Marcos 2007

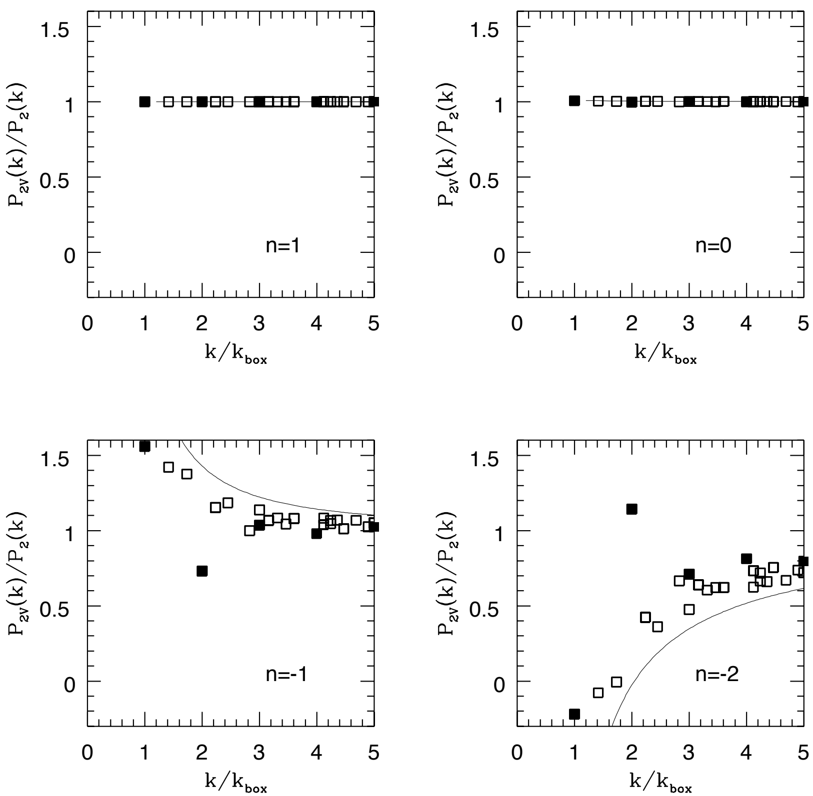
Anisotropy at large scales due to finite periodic box effects

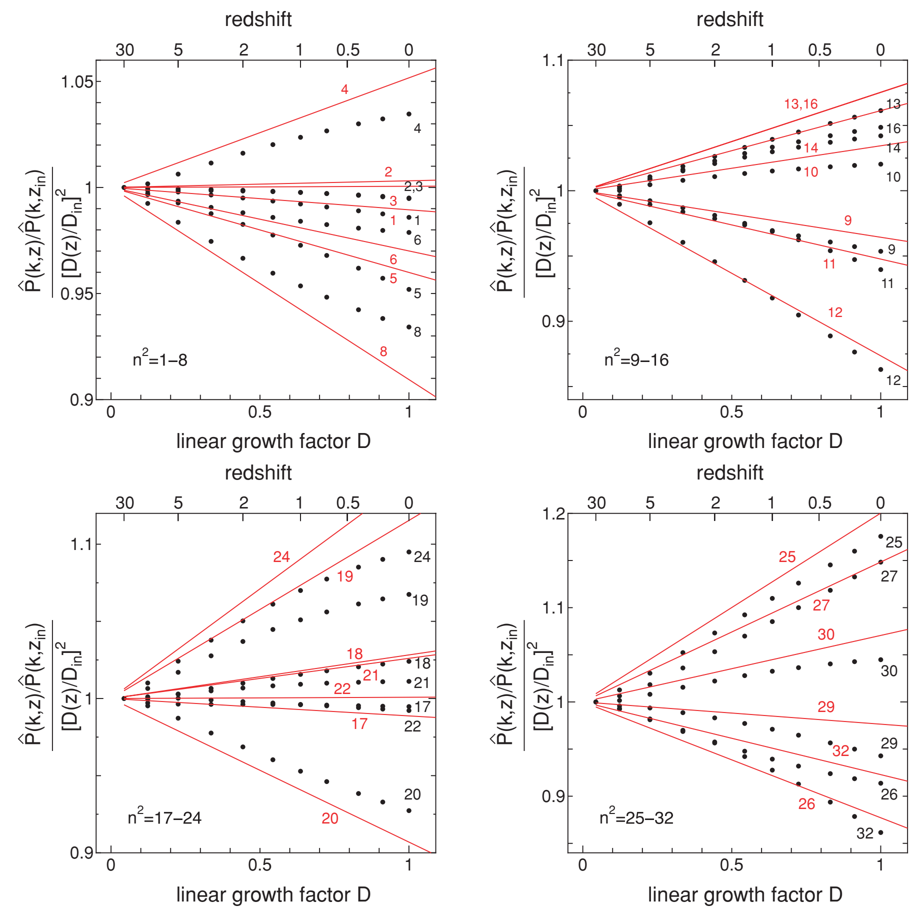
$$\delta^{N\text{-body}}_{\mathbf{k},n}(z) = \delta^{\mathrm{L}}_{\mathbf{k},n}(z) + \delta^{(2)}_{\mathbf{k},n}(z) + \ldots.$$

$$\hat{P}^{\mathrm{PT}}(k_i, z) \equiv \left\langle \left|\delta^{\mathrm{PT}}_{\mathbf{k},n}\right|^2 \right\rangle_i ,$$

$$\left|\delta^{\mathrm{PT}}_{\mathbf{k},n}\right|^2 \equiv \left|\delta^{\mathrm{L}}_{\mathbf{k},n}(z)\right|^2 + 2\Re\left[\delta^{\mathrm{L}}_{\mathbf{k},n}(z)\delta^{(2)}_{-\mathbf{k},n}(z)\right]$$

does not vanish when number of modes is finite



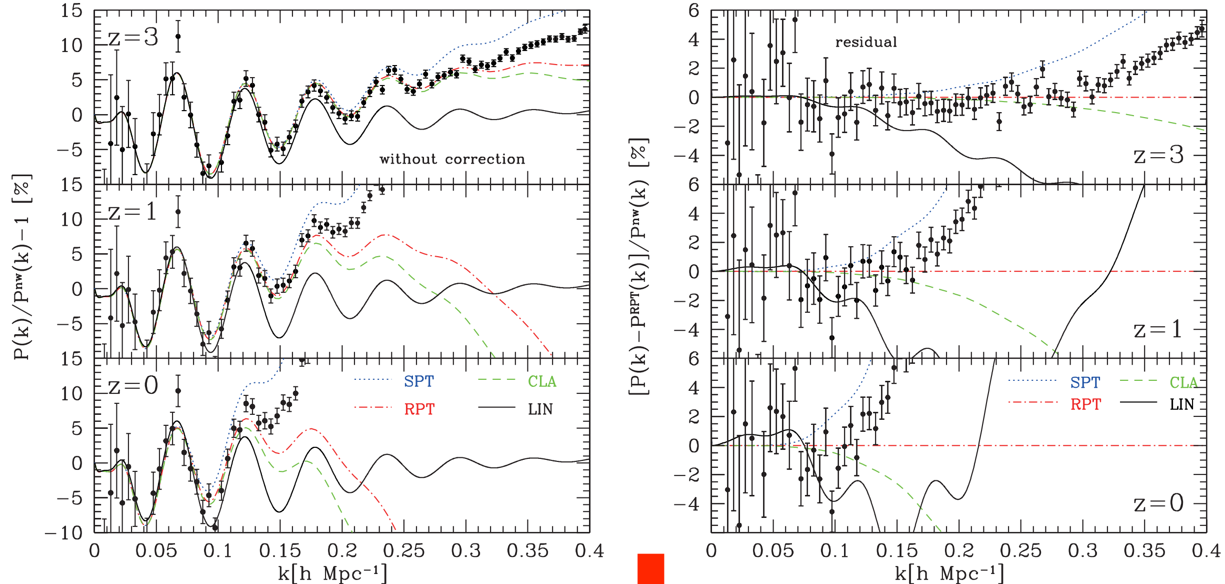


2nd order power-spectrum: finite box versus infinite

Seto 1999, ApJ 523, 24

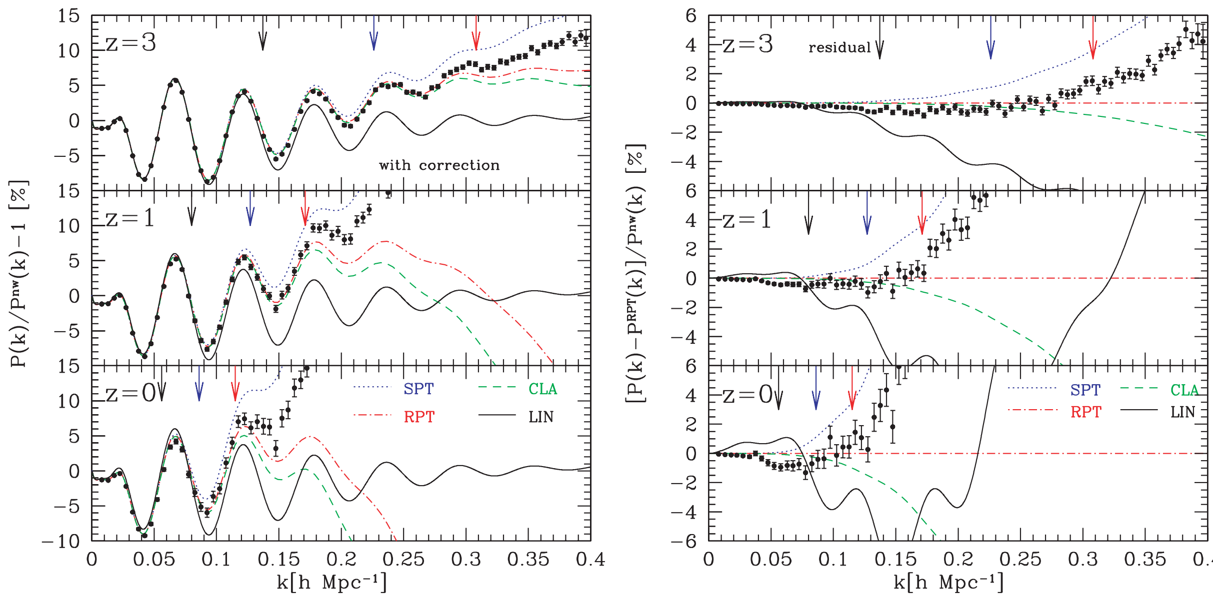
Takahashi et al., 2008, MNRAS 389, 1675

Anisotropy at large scales can be corrected



Nishimichi et al. 2009, PASJ 61, 321

Correction at 2nd order



Cosmological N-body simulations with suppressed variance

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18 March 2016

Fixing large sampling variance at small k

Angulo & Pontzen 2016, arXiv:1603.05253

ABSTRACT
We present and test a method that dramatically reduces variance arising from the sparse sampling of wavemodes in cosmological simulations. The method uses two simulations which are *fixed* (the initial Fourier mode amplitudes are fixed to the ensemble average power spectrum) and *paired* (with initial modes exactly out of phase). We measure the power spectrum, monopole and quadrupole redshift-space correlation functions, halo mass function and reduced bispectrum at $z = 1$. By these measures, predictions from a fixed pair can be as precise on non-linear scales as an average over 50 traditional simulations. The fixing procedure introduces a non-Gaussian correction to the initial conditions; we give an analytic argument showing why the simulations are still able to predict the mean properties of the Gaussian ensemble. We anticipate that the method will drive down the computational time requirements for accurate large-scale explorations of galaxy bias and clustering statistics, enabling more precise comparisons with theoretical models, and facilitating the use of numerical simulations in cosmological data interpretation.

Key words: N-body simulations, large-scale structure of the Universe

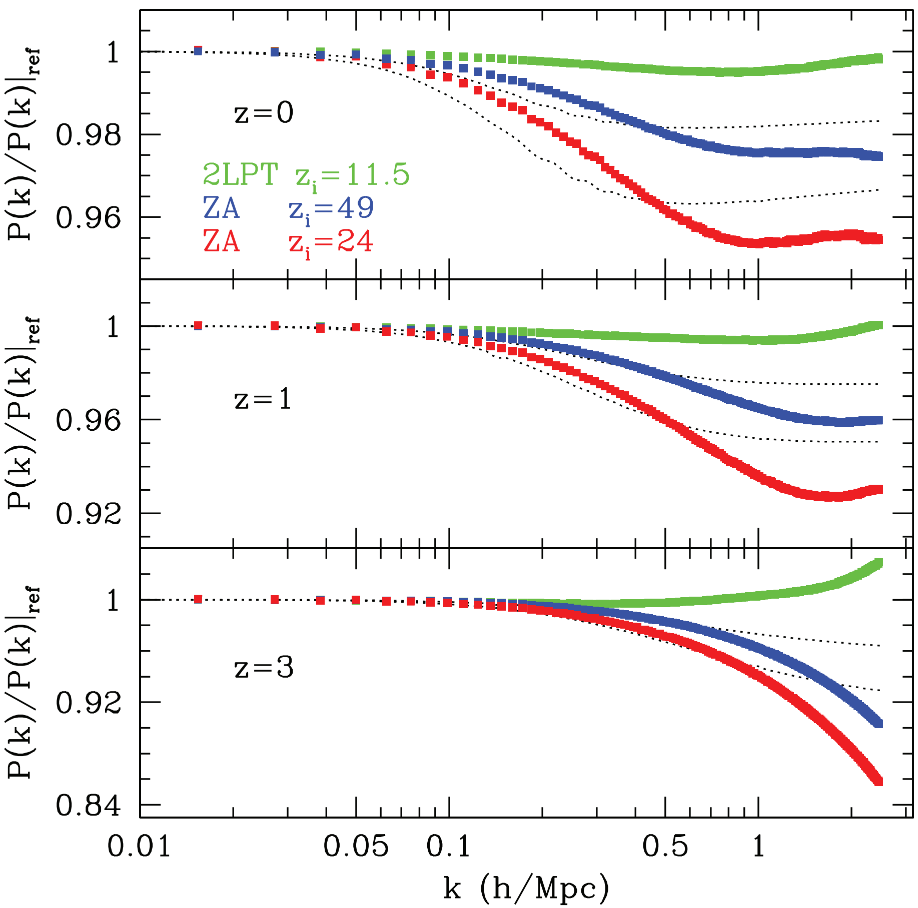
Reducing sampling variance by fixing the modulus of the fourier mode while keeping its phase random. False good idea or quite handy ?

In my opinion :

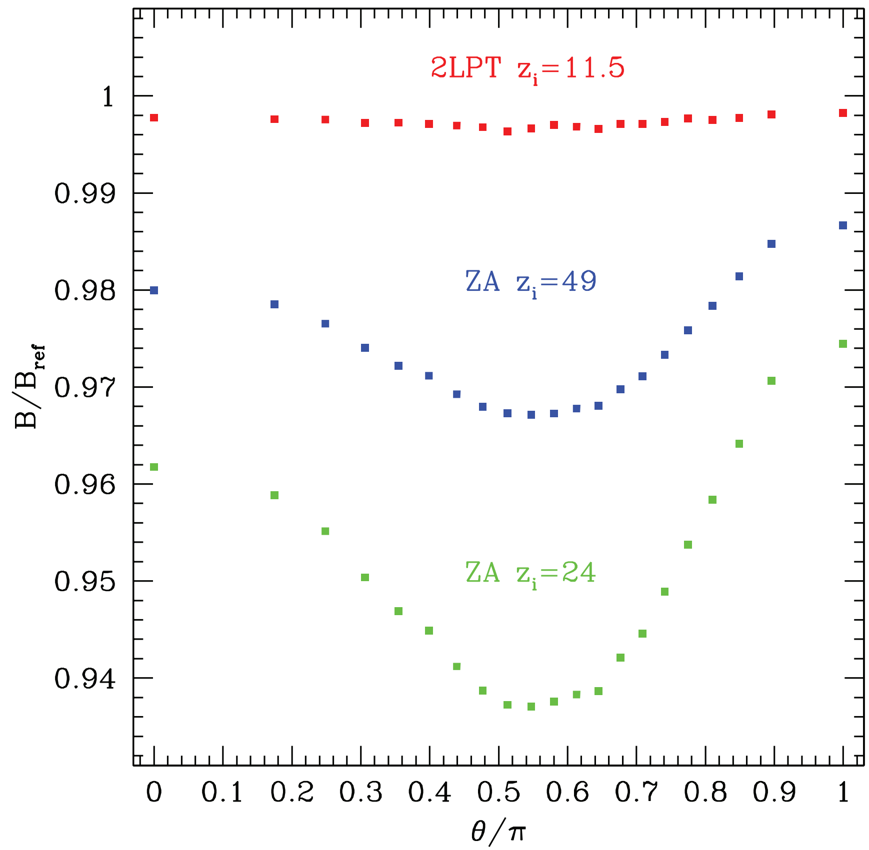
- Seems hazardous as the initial conditions are actually non Gaussian even though a pairing technique is used to reduce this effect
- Does not reduce other finite box effects e.g. anisotropy induced by discrete sampling of k space
- Perturbation theory can potentially be tested by taking into account the noise e.g. on the effective noisy power-spectrum
- Cosmic variance remains useful to have for mock catalogs

``Transcients due to Zeldovich approximation

Crocce et al. 2006



Power-spectrum



Bispectrum

Advertisement:

ColDICE: a parallel Vlasov-Poisson solver using moving adaptive simplicial tessellation

(so without particles)

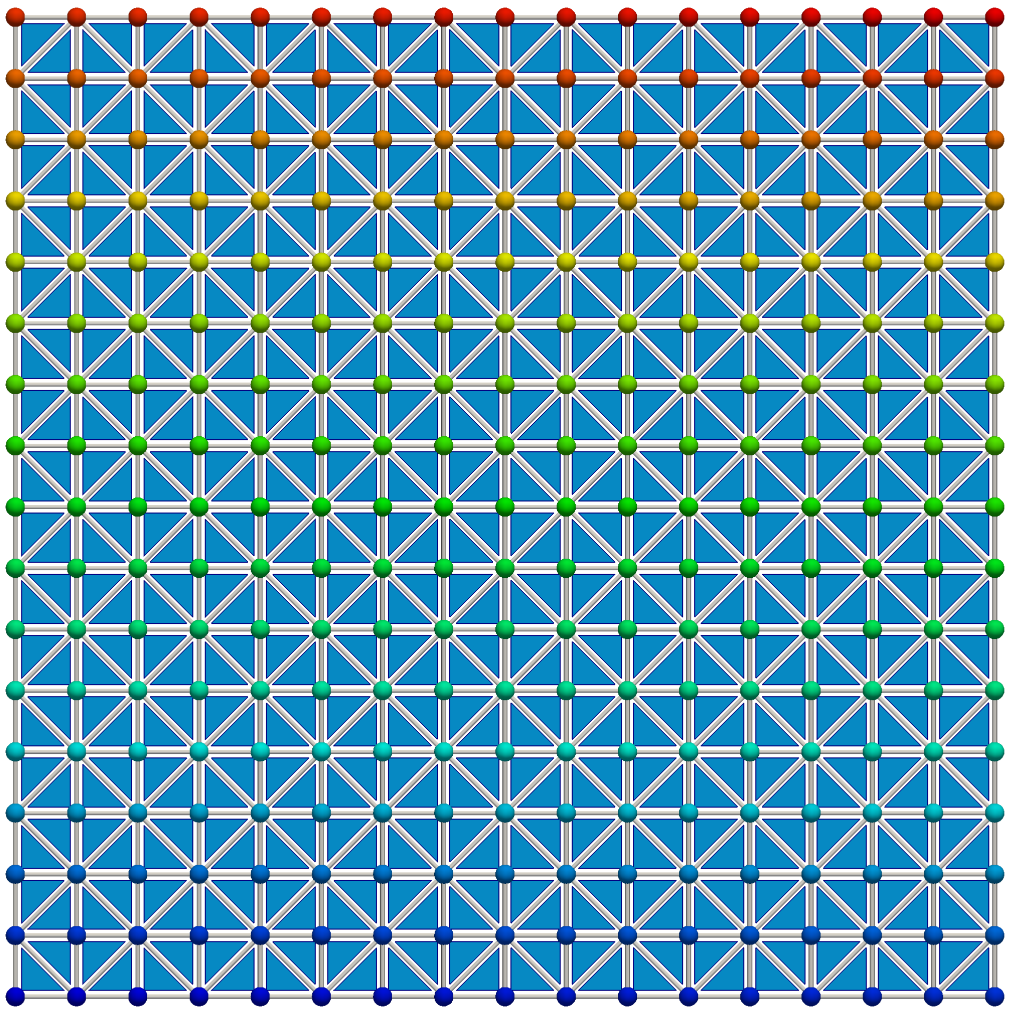
S. Colombi (IAP)

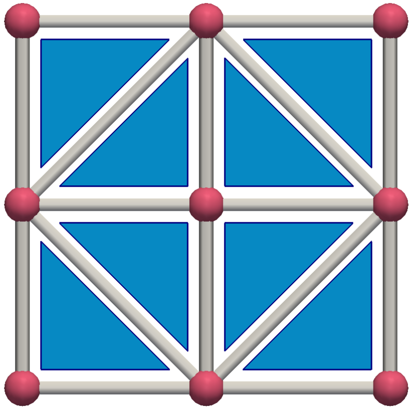
&

T. Sousbie (IAP & The Univ. of Tokyo)

Sousbie & Colombi 2016, accepted in JCP, arXiv:1509.07720

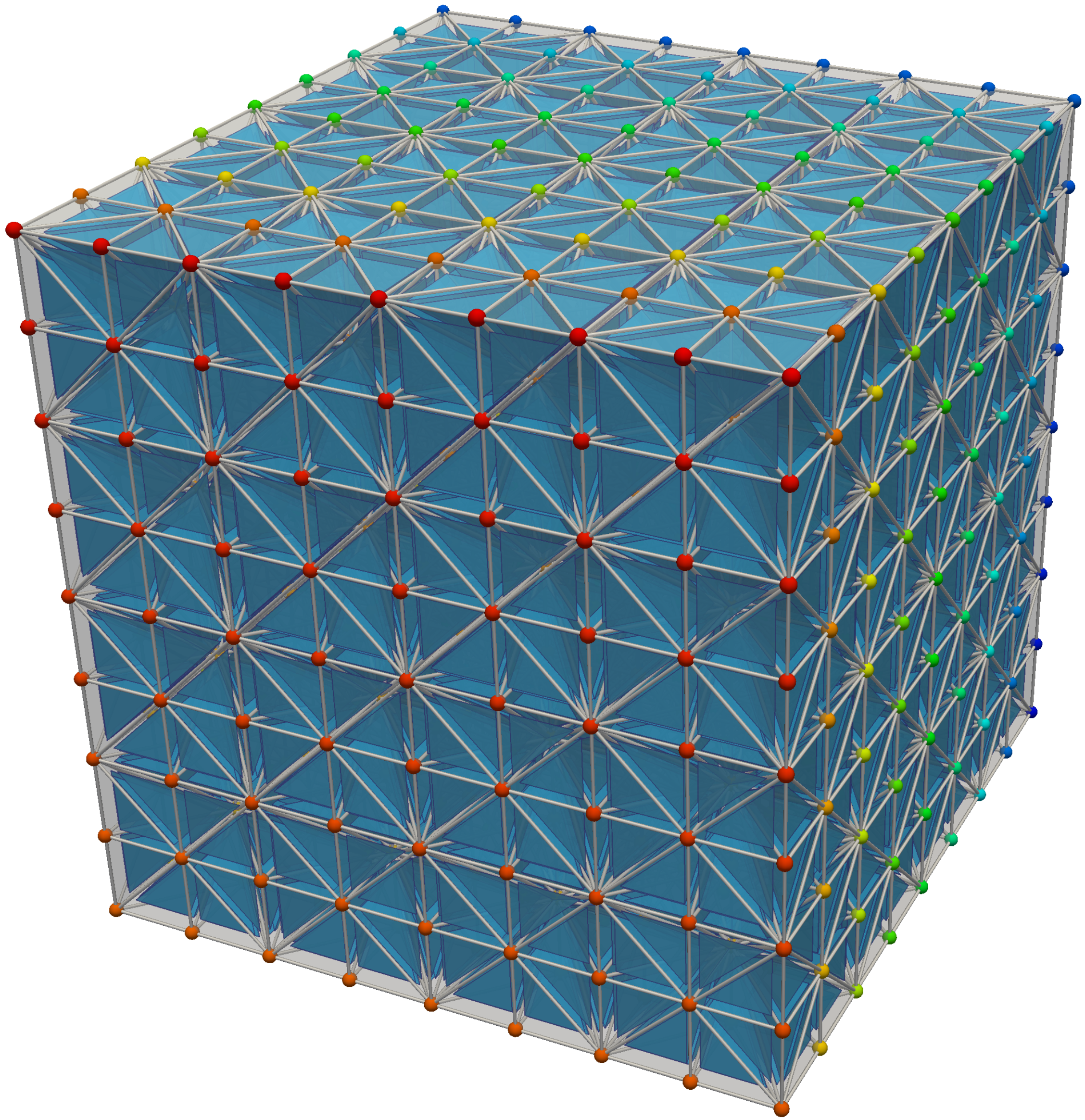
Phase-space sheet tessellation: 2D

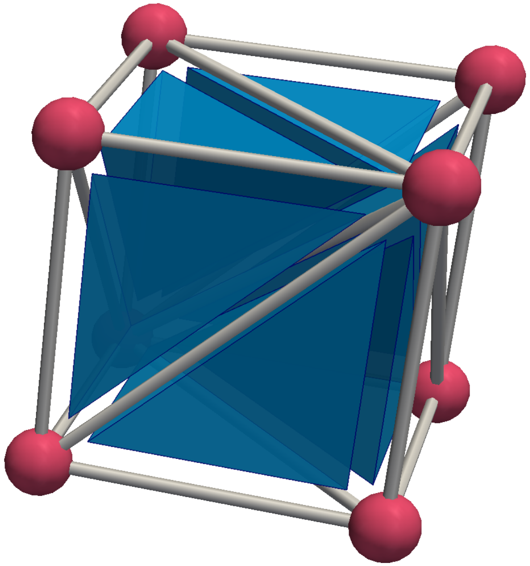


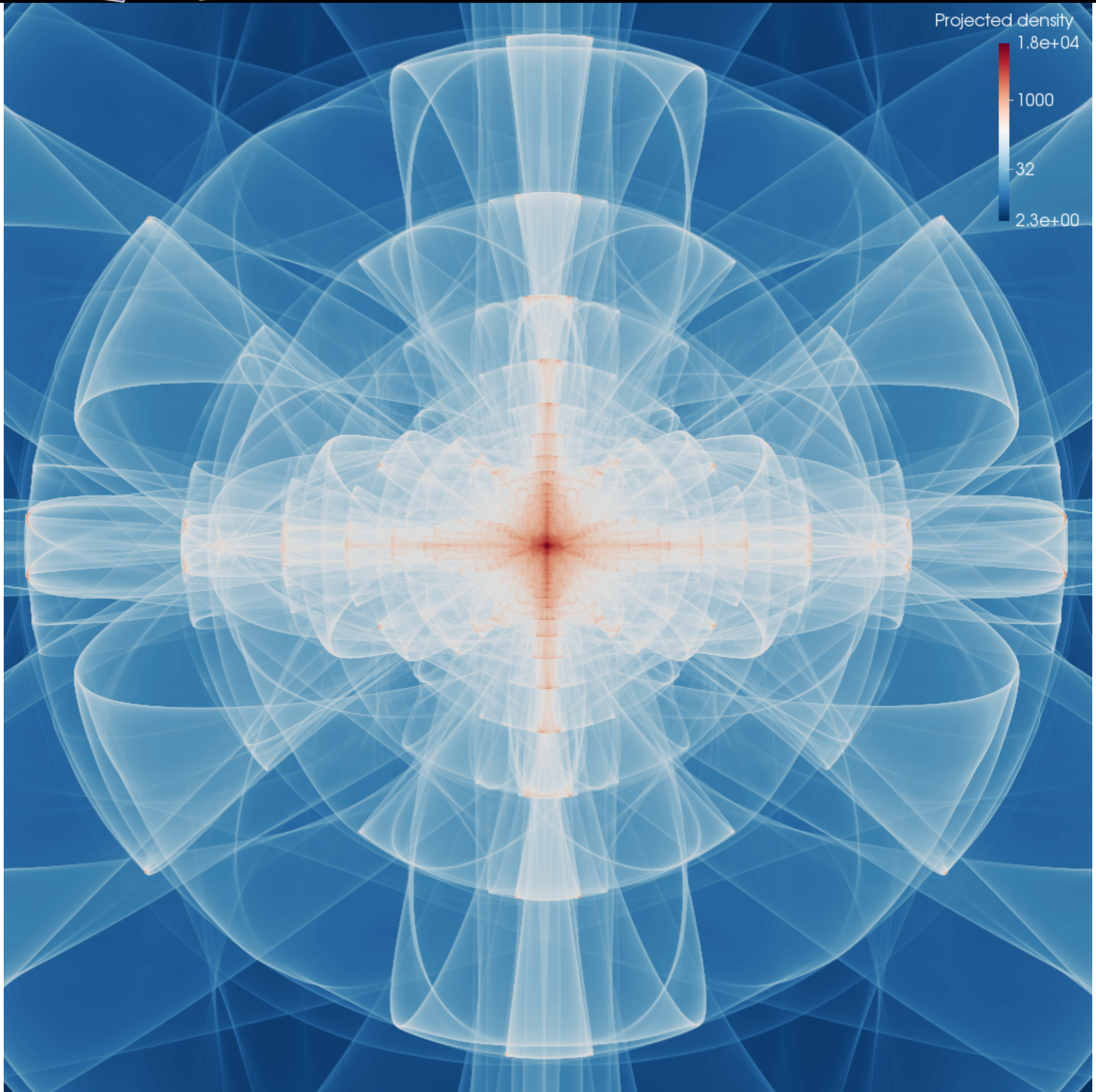


Abel, Hahn & Kaehler 2012; Shandarin, Habib & Heitmann 2012; Hahn, Abel & Kaehler 2013; Hahn & Angulo 2016; Sousbie & Colombi 2016

Phase-space sheet tessellation: 3D

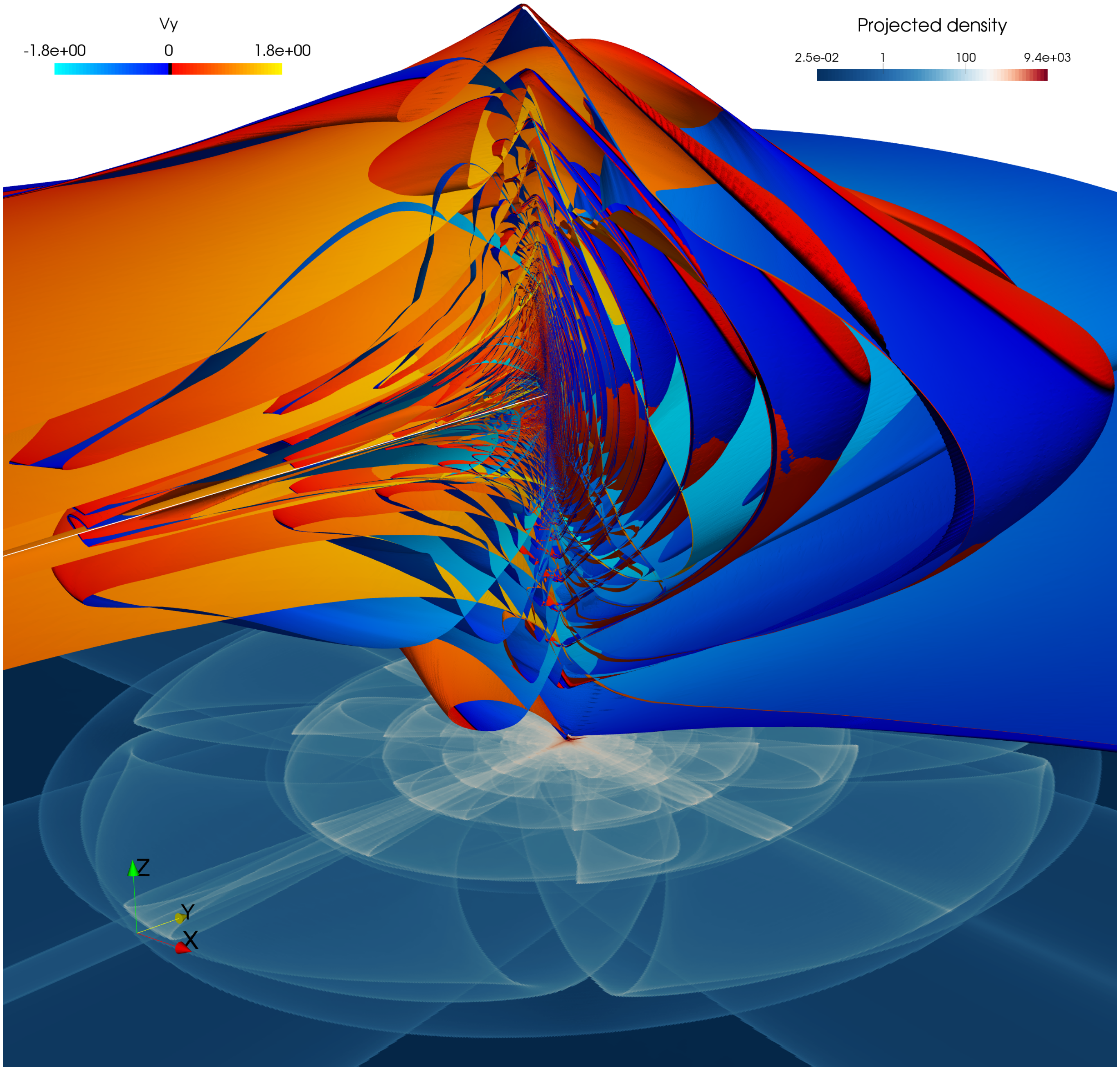






Sousbie & Colombi 2016

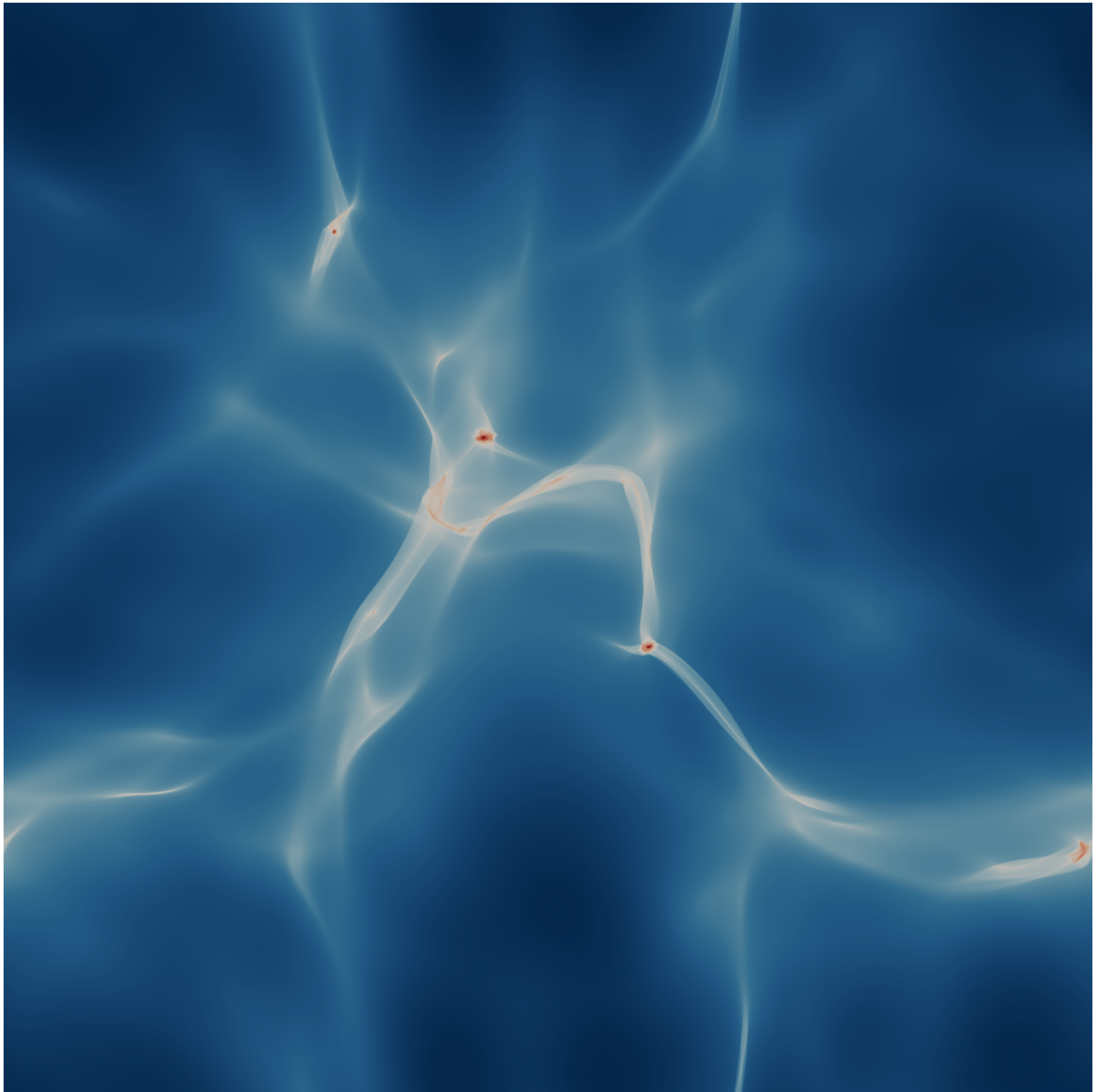
www.vlasix.org



(x,y,v_x) space, color v_y

Sousbie & Colombi 2016

Cosmological warm dark matter simulation in 3 (6D phase-space)



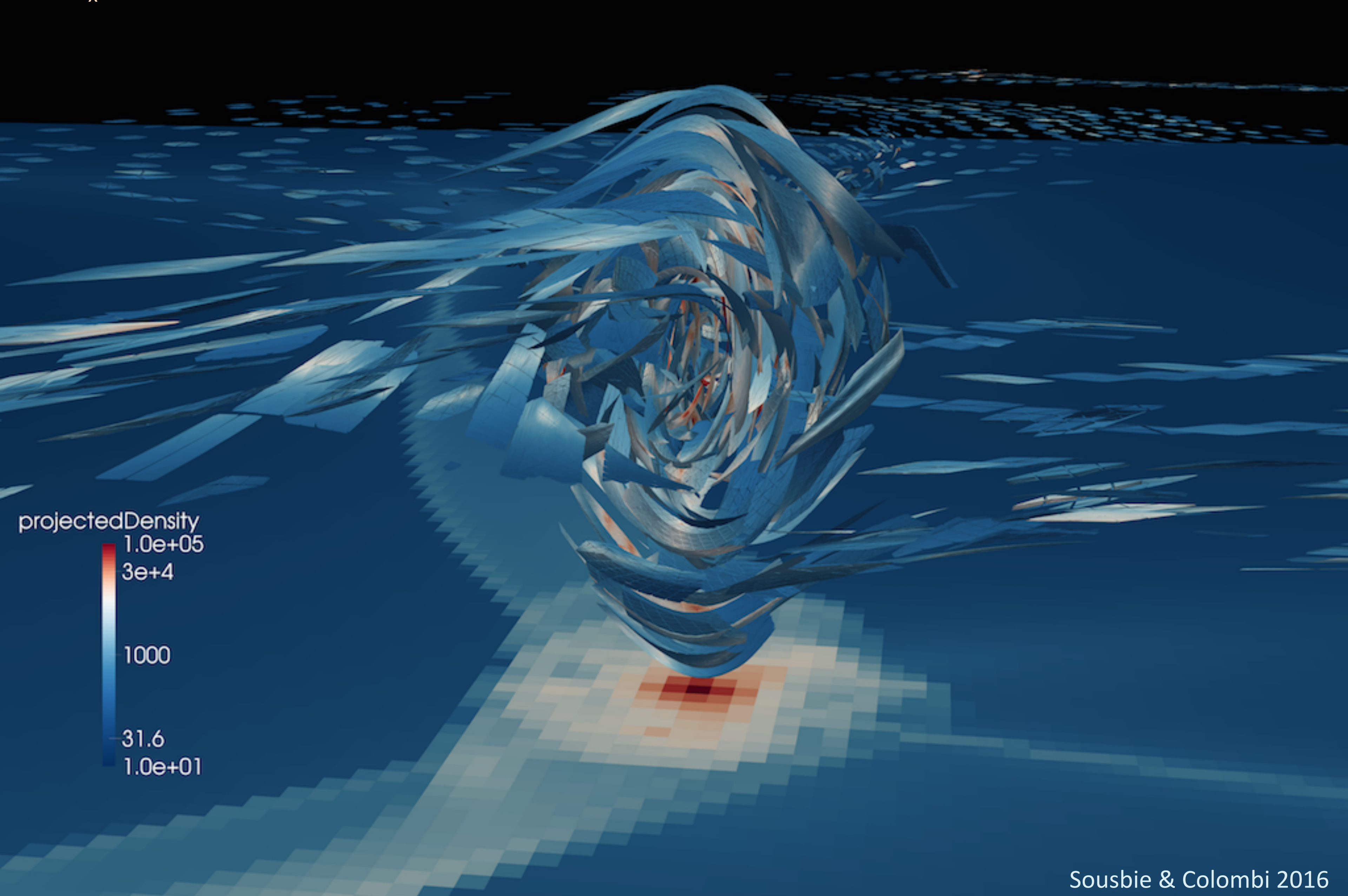
Projected density

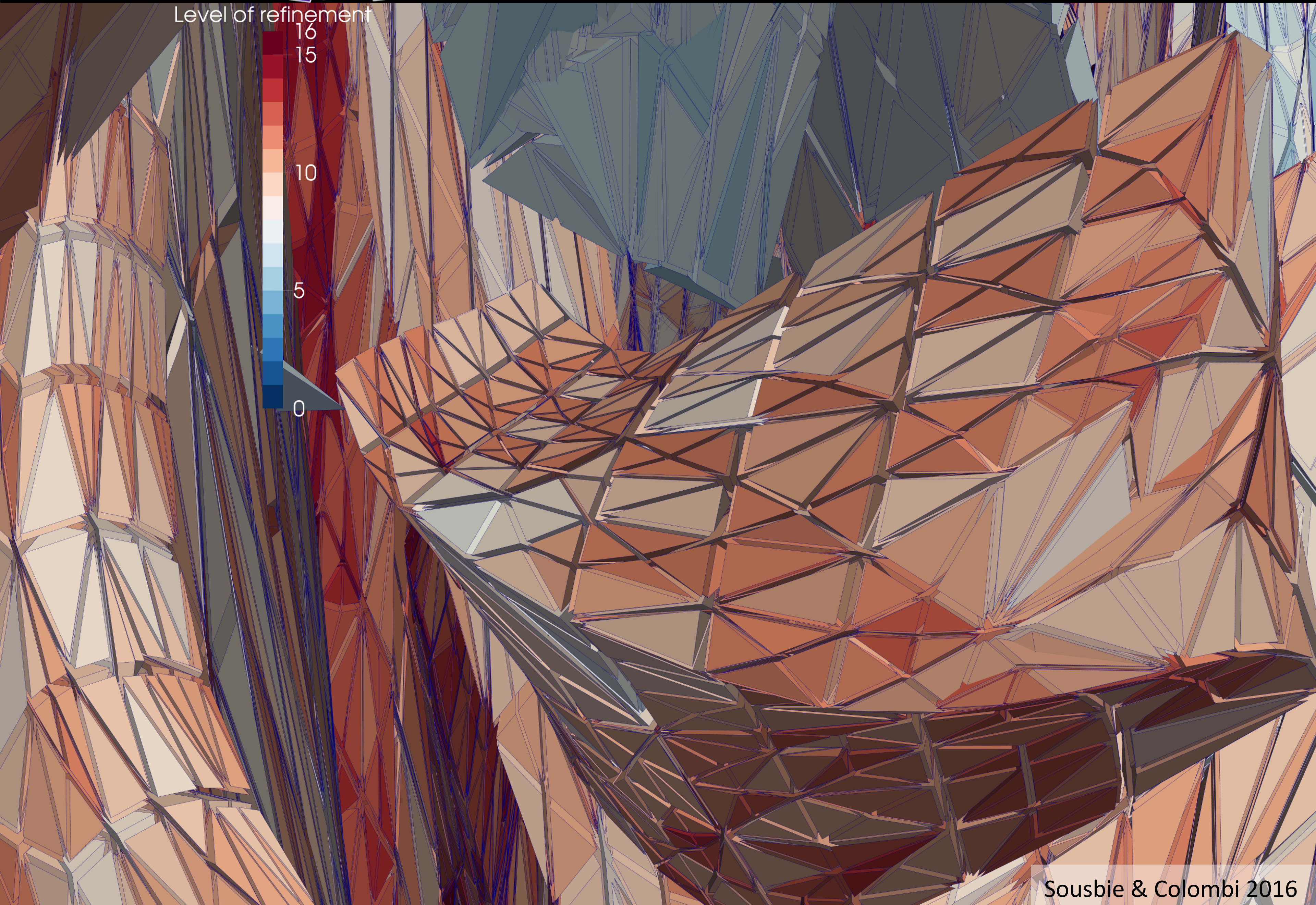
Sousbie & Colombi 2016

VLA6

www.vlasix.org

(x,y,v_x) subspace





Sousbie & Colombi 2016