EFT of LSS at NNLO

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[with Katelin Schutz, Mikhail Solon, Jon Walsh, and Kathryn Zurek]

Theory Meets Expectations - Paris
May 26, 2016

This talk

[DB, Schutz, Solon, Walsh, Zurek, arXiv:1512.07630 (PRD);
DB, Schutz, Solon, Zurek, arXiv:1604.01770]

- Eulerian EFT at NNLO at one loop
- Trispectrum and covariance [see Katelin's talk]
- FnFast
- Few words on squeezed limits

The Cosmology Frontier

seeking for precision

Want to use LSS to measure effects such as neutrino mass, primordial non-Gaussianities, DE properties, etc.. On large scales these are tiny effects and non-linear gravitational dynamics acts as a background

One needs precise predictions for the fiducial power spectrum and its covariance [See Uros' and Pier-Stefano's talks]

Perturbation Theory

dark matter as a perfect fluid

$$\frac{\partial \rho}{\partial \tau} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} + \nabla \Phi = 0$$

$$\delta(\mathbf{k}, \tau) = \sum_{n=1}^{\infty} D^n(\tau)\, \delta^{(n)}(\mathbf{k})$$

$$\delta^{(n)}(\mathbf{k}) = \int \frac{d^3 q_1}{(2\pi)^3} \ldots \frac{d^3 q_n}{(2\pi)^3} F_n(\mathbf{q}_1, \ldots, \mathbf{q}_n)(2\pi)^3 \delta_D\left(\mathbf{k} - \sum_{i=1}^{n} \mathbf{q}_i\right) \delta^{(1)}(\mathbf{q}_1) \ldots \delta^{(1)}(\mathbf{q}_n)$$

+ similar expansion for the velocity

Perturbation Theory

(unphysical) UV contributions

- Beyond tree-level there are loop integrals. Integrate over short scales where perturbation theory (and the perfect fluid description) cease to be applicable

- As in QFT, EFT provides a way to 'renormalize' these unphysical contributions [Baumann, Nicolis, Senatore, Zaldarriaga (2012); Carrasco, Hertzberg, Senatore (2012); etc…]

- Errors would also be not well-defined
[e.g. Baldauf, Mirbabayi, Simonovic, Zaldarriaga (2016)]

EFT of **LSS**

the Wilsonian approach

Integrate out short-modes and re-derive fluid equations
Long-wavelength modes behave as an imperfect fluid

$$\frac{\partial \rho_l}{\partial \tau} + \nabla \cdot (\rho_l \mathbf{v}_l) = \nabla \cdot \sigma_{\text{heat}}(\rho_l, \mathbf{v}_l, \tau)$$

$$\frac{\partial \mathbf{v}_l}{\partial \tau} + \mathcal{H}\mathbf{v}_l + \mathbf{v}_l \cdot \nabla \mathbf{v}_l = \frac{1}{\rho_l} \partial \tau_{\text{stress}}(\rho_l, \mathbf{v}_l, \tau) + f(\sigma_{\text{heat}})$$

where $\pi_l = \rho_l \mathbf{v}_l + \sigma_{\text{heat}}$

EFT of **LSS**

bottom-up

- Construct EFT sources using smoothed fields and according to IR symmetries: conservation of mass, rotational and Galilean invariance

- Organize EFT sources as an expansion in perturbations and derivatives

- Do perturbation theory

EFT of LSS

Application of EFT to perturbative calculations

- One-loop power spectrum [Carrasco, Hertzberg, Senatore (2012)]

- Two-loop power spectrum [Carrasco, Foreman, Green, Senatore (2014); Baldauf, Mercolli, Zaldarriaga (2015)]

- One-loop bispectrum [Angulo, Foreman, Schmittfull, Senatore (2015); Baldauf, Mercolli, Mirbabayi, Pajer (2015)]

- Lagrangian formulations [Porto, Senatore, Zaldarriaga (2014); Vlah, White, Aviles (2015)]

EFT at **NNLO**

[odds and ends]

- Heat conduction terms and vorticity
- Building blocks
- Non-locality-in-time and convective derivatives
- Minimal basis of operators

EFT at **NNLO**

heat conduction terms and vorticity

$$\frac{\partial \rho_l}{\partial \tau} + \nabla \cdot (\rho_l \mathbf{v}_l) = \underline{\nabla \cdot \sigma_{\mathrm{heat}}}(\rho_l, \mathbf{v}_l, \tau)$$

can be reabsorbed into a redefinition of the velocity

[Carrasco, Foreman, Green, Senatore (2014); Mercolli, Pajer (2015); Abolhasani, Mirbabayi, Pajer (2015)]

$$\mathbf{v}_l = \mathbf{v}_\pi + \sigma_{\mathrm{heat}}/\rho_l$$

Density correlators and their EFT counter-terms are independent of this redefinition

But, in the new velocity basis, vorticity must be included

EFT at **NNLO**

equations of motion

$$\frac{\partial \rho_l}{\partial \tau} + \nabla \cdot (\rho_l \mathbf{v}_\pi) = 0$$

$$\frac{\partial \mathbf{v}_\pi}{\partial \tau} + \mathcal{H}\mathbf{v}_\pi + \mathbf{v}_\pi \cdot \nabla \mathbf{v}_\pi = \frac{1}{\rho_l} \partial \tau_{\mathbf{stress}}$$

Vorticity $\omega = \nabla \times \mathbf{v}_\pi$ is sourced by the curl of the stress tensor starting at NLO and it feeds back into the continuity and the $\boldsymbol{\theta}$-Euler equation at NNLO

EFT at **NNLO**

building blocks

Galilean-invariant building blocks:

$$\{\partial_i\partial_j\phi, \partial_i\partial_j\phi_v(\equiv \partial_i v_j), D_\tau, \partial_k\}$$

Construct stress-tensor from all independent contractions up to three fields

EFT at **NNLO**

time non-locality and convective derivatives

Integration of short-scales generates a non-local-in-time stress-tensor

$$\tau_{ij}(\mathbf{x},\tau) = \int \mathrm{d}\tau' K(\tau,\tau') \underline{\bar{\tau}_{ij}}(\mathbf{x}_{\mathrm{fl}},\tau')$$

in a Eulerian framework can be expanded as

$$\bar{\tau}_{ij}(\mathbf{x}_{\mathrm{fl}},\tau') = \bar{\tau}_{ij}(\mathbf{x},\tau') - \partial_k \bar{\tau}_{ij}(\mathbf{x},\tau') \int_{\tau'} \mathrm{d}\tau'' v^k(\mathbf{x},\tau'') \\ + \partial_k \bar{\tau}_{ij}(\mathbf{x},\tau') \int_{\tau'} \mathrm{d}\tau'' \partial_b v^k(\mathbf{x},\tau'') \int_{\tau''} \mathrm{d}\tau''' v^b(\mathbf{x},\tau''') + \dots$$

using $\mathbf{x}_{\mathrm{fl}}(\tau,\tau') = \mathbf{x} - \int_{\tau'}^{\tau} \mathrm{d}\tau'' \mathbf{v}(\mathbf{x}_{\mathrm{fl}}(\tau,\tau''),\tau'')$

and unknown kernels can be reabsorbed order-by-order into EFT coefficients

Finally, the stress-tensor can be rewritten in terms of convective derivatives acting on local operators

$$\tau_{ij}(\mathbf{x},\tau) = \sum_{n=1}^{\infty} d_n \, D_\tau^n \bar{\tau}_{ij}(\mathbf{x},\tau)$$

EFT at NNLO

time non-locality and convective derivatives

Convective derivatives can be shown to be redundant through NNLO. Thus, one can construct the stress-tensor using the local building-blocks

$$\{\partial_i\partial_j\phi, \partial_i\partial_j\phi_v\}$$

$$k_i\tau^{ij} = \bar{c}_s^{\delta} k^j \delta(\mathbf{k}) + \frac{\bar{c}_s^{\theta}}{\mathcal{H}f} k^j \theta(\mathbf{k}) \quad \leftarrow \text{LO}$$

$$+ \int d\mathbf{q} \sum_{n=1}^{4} \left[\bar{c}_n^{\delta\delta}\delta(\mathbf{q})\delta(\mathbf{k}-\mathbf{q}) + \frac{\bar{c}_n^{\theta\theta}}{\mathcal{H}^2 f^2}\theta(\mathbf{q})\theta(\mathbf{k}-\mathbf{q}) + \frac{\bar{c}_n^{\delta\theta}}{\mathcal{H}f}\delta(\mathbf{q})\theta(\mathbf{k}-\mathbf{q}) + \frac{\bar{c}_n^{\theta\delta}}{\mathcal{H}f}\theta(\mathbf{q})\delta(\mathbf{k}-\mathbf{q})\right] k_i e_n^{ij}(\mathbf{q}, \mathbf{k}-\mathbf{q}) \quad \leftarrow \text{NLO}$$

$$\text{NNLO} \rightarrow \; + \int d\mathbf{q}_1 d\mathbf{q}_2 \sum_{n=1}^{10} \bar{c}_n^{\delta\delta\delta}\delta(\mathbf{q}_1)\delta(\mathbf{q}_2)\delta(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2) k_i E_n^{ij}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)$$

EFT at **NNLO**

counting independent operators

Once inserted in the equations of motion we find

- 1 (LO) + 3 (NLO) + 8 (NNLO) independent EFT operators for a generic trispectrum configuration $T(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$

- 1 (LO) + 3 (NLO) + 3 (NNLO) independent EFT operators for a covariance configuration $\langle T(\mathbf{k}_1, -\mathbf{k}_1, \mathbf{k}_2, -\mathbf{k}_2)\rangle_{\text{angle}}$

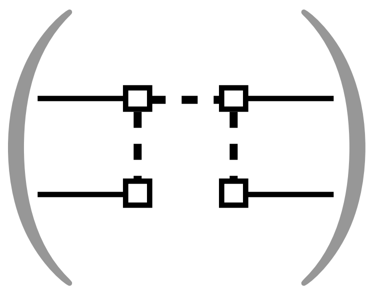
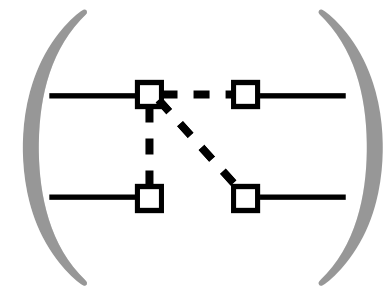
[EFT operators can be further reduced, see Katelin's talk]

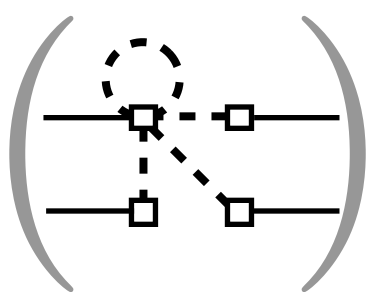
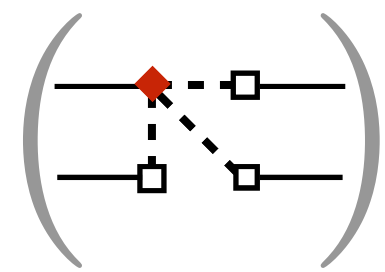
SPT and **EFT** at **NNLO**

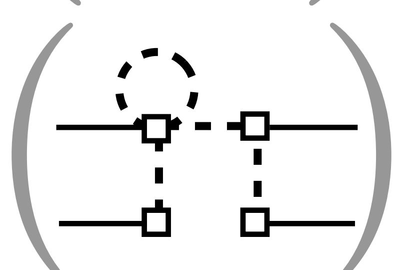
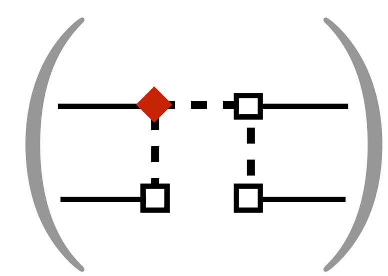
one-loop trispectrum

Solve equations of motion, derive effective kernels, and calculate 4-point function tree-level + one-loop diagrams

$$\delta(\mathbf{k},\tau) = \sum_{n=1}^{\infty} \left[D^n(\tau)\,\delta^{(n)}(\mathbf{k}) + \epsilon\, D^{n+2}(\tau)\tilde{\delta}^{(n)}(\mathbf{k}) \right]$$

$T = \langle \delta^{(2)}\delta^{(2)}\delta^{(1)}\delta^{(1)} \rangle$  $+ \langle \delta^{(3)}\delta^{(1)}\delta^{(1)}\delta^{(1)} \rangle$ 

$+ \langle \delta^{(5)}\delta^{(1)}\delta^{(1)}\delta^{(1)} \rangle$  $+ \langle \tilde{\delta}^{(3)}\delta^{(1)}\delta^{(1)}\delta^{(1)} \rangle$ 

$+ \langle \delta^{(4)}\delta^{(2)}\delta^{(1)}\delta^{(1)} \rangle$  $+ \langle \tilde{\delta}^{(2)}\delta^{(2)}\delta^{(1)}\delta^{(1)} \rangle$  $+ \ldots$

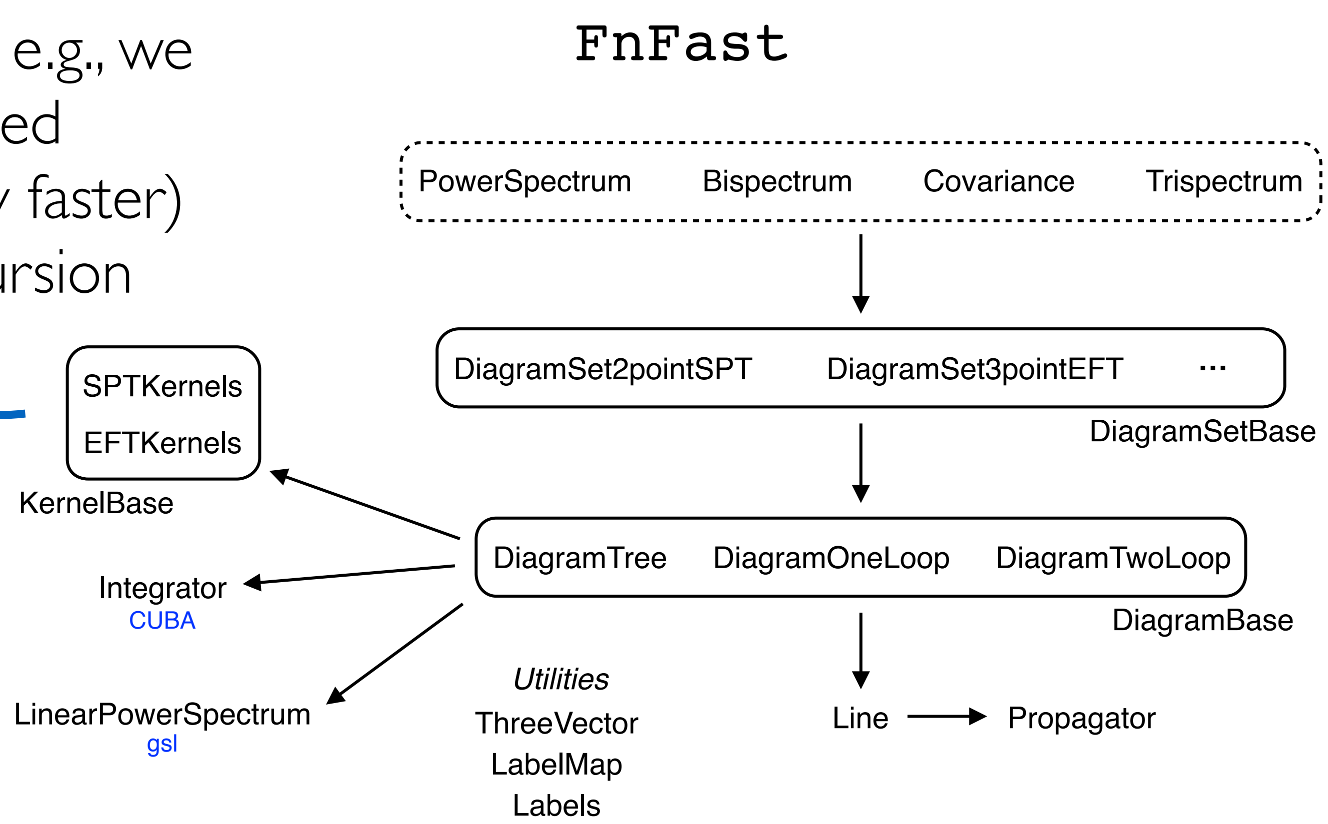
FnFast

turning the crank

https://github.com/jrwalsh1/FnFast

The idea is to develop a fast and flexible framework to perform LSS perturbative calculations. E.g., in SPT, EFT, LPT, regPT, etc..

For SPT kernels, e.g., we employ a modified (computationally faster) form of the recursion relations



Diagrams (including symmetry factors and IR reg) are built in terms of 'abstract' kernels and propagators.
These are specified (e.g., SPT or EFT kernels) only later, when the actual evaluation, including integrations, takes place

Squeezed limits

[work in progress with Mikhail Solon]

There has been lot of work on squeezed limits of LSS N-point functions

[Kehagias, Perrier, Riotto; Valageas; Peloso, Pietroni; Creminelli, Norena, Simonovic, Vernizzi; Ben-Dayan, Konstandin, Porto, Sagunski; Pajer, Schmidt, Zaldarriaga; Wagner, Schmidt, Chiang, Komatsu, etc…]

Squeezed covariance as a response function $C(k,q) \xrightarrow{q \ll k} P_{\rm lin}^2(q) P(k) \mathcal{R}(k)$

Response of the power spectrum to long-wavelength backgrounds

$\langle T(\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}_1, \mathbf{q}_2) \rangle \xrightarrow{q_{1,2} \ll k_{1,2}} P_{\rm lin}^2(q) P(k) \mathcal{R}_{\rm iso}(k)$ [Wagner, Schmidt, Chiang, Komatsu (2015)]

$\mathcal{R}(k) = \mathcal{R}_{\rm iso}(k) + \mathcal{R}_{\rm tidal}(k)$

Because of the incomplete angular average, pick up terms from the tidal response too

Response functions can be measured precisely, and could provide a better way to measure EFT coefficients

Summary

- One-loop calculation of 4-point correlators in SPT and EFT
- Get results for trispectrum and covariance of the matter-spectrum
- FnFast, platform for automated numerical evaluations

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[Thanks!]