Oscillations in the CMB bispectrum

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Primordial Non-Gaussianities

- The primordial curvature field Φ describes the initial anisotropies.
- The 2-point function $\langle \Phi(\mathbf{x}_1)\Phi(\mathbf{x}_2)\rangle$ and 3-point function $\langle \Phi(\mathbf{x}_1)\Phi(\mathbf{x}_2)\Phi(\mathbf{x}_3)\rangle$ can be calculated from the inflation action.
- E.g. single field slow roll inflation

$$B_\Phi(k_1,k_2,k_3) \approx \frac{N}{(k_1k_2k_3)^2}\left((4\epsilon-2\eta)\mathcal{S}^{local}(k_1,k_2,k_3)+\frac{5}{3}\epsilon\mathcal{S}^{equil}(k_1,k_2,k_3)\right)$$

- The bad news: NG in single field slow roll inflation is unobservably small.
- But: The algebraically simplest model is not necessarily the most realistic one. Inflationary Lagrangians derived from string theory or from EFT considerations often predict significant NG.

Non-Gaussianity in the CMB

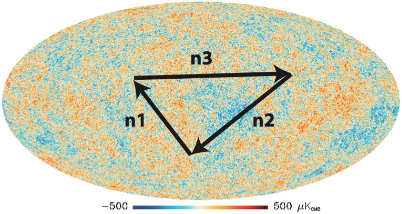
How to calculate the CMB anisotropy from the primordial anisotropy?

- The CMB is the most straight forward (linear) way to measure primordial NG (but not necessarily the most powerful)
- A given primordial curvature field $\Phi(x)$ induces a CMB temperature anisotropy $T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$ with

$$a_{lm} \propto \int d^3\mathbf{k}\ \Phi(k)\ T_{rad}(k)\ Y_{lm}(\hat{\mathbf{k}})$$

- From $\langle \Phi(\mathbf{x}_1)\Phi(\mathbf{x}_2)\Phi(\mathbf{x}_3)\rangle$ we can thus calculate the CMB bispectrum

$$B^{m_1 m_2 m_3}_{l_1 l_2 l_3} = \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle$$



Oscillations in the primordial bispectrum

A special class of non-gaussianities: Oscillations

- The inflaton potential may contain steps or periodic features. For example

$$V(\varphi) = \frac{1}{2} m^2 \varphi^2 \left[1 + \tanh\left(\frac{\varphi - \varphi_s}{d} \right) \right]$$

- The feature forces the inflaton away from the attractor solution, and induces oscillations in the power spectrum and bispectrum as it relaxes back.

- A simple and generic shape is

$$B_{\Phi}^{\mathrm{osci}}(k_1, k_2, k_3) = \frac{f_{NL}}{(k_1 k_2 k_3)^2} \sin\left(\frac{k_1 + k_2 + k_3}{3 k_c} + \phi \right)$$

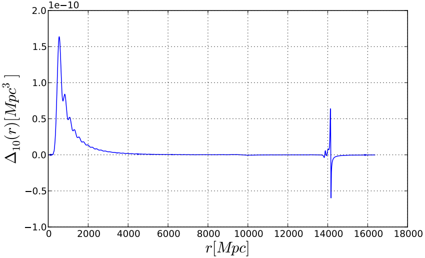
  This shape is factorizable.

- Several inflation models predict oscillations. They have not been constrained at high oscillation frequency.

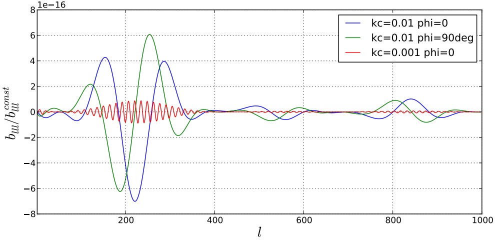
Oscillations in the CMB

- To calculate the CMB bispectrum we convolve the primordial potential with the transfer functions.

  Example of transfer function $\rightarrow$



- The resulting bispectrum is here plotted for $l_1 = l_2 = l_3$



The KSW estimator

How to measure non-gaussianity in the CMB?

- Goal: Estimate the amplitude f_{NL} of a given theoretical bispectrum $B^{theo}_{l_1 l_2 l_3}$ in the CMB map.
- The optimal signal-to-noise estimator is

$$\hat{f}_{NL} = \frac{1}{\mathcal{N}} \sum_{l_1, l_2, l_3} \frac{B^{theo}_{l_1 l_2 l_3} B^{obs}_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}}$$

- This estimator is computationally extremely costly. It can be simplified if B can be factorized as $B_{l_1, l_2, l_3} = (X_{l_1} Y_{l_2} Z_{l_3}) + \mathrm{perm.}$
- A number of factorizable bispectra has been searched with this estimator (WMAP, Planck). No primordial bispectrum has been found. However it is easy to miss something with model dependent estimators!

The KSW estimator for oscillations

- The (simplified) KSW estimator for oscillations is

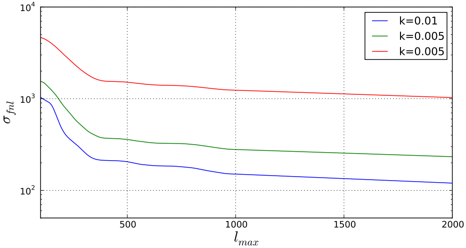
$$\hat{f}_{NL} = \frac{1}{\mathcal{N}} \int r^2 dr \int d\Omega \; M_X^3(r, \hat{n})$$

  with

$$M_X(r, \hat{n}) = \sum_{lm} (C^{-1}a)_{lm} X_l(r) Y_{lm}(\hat{n})$$

  where $X_l(r)$ is a filter tailored to the oscillating bispectrum.

- Precision forecast on f_{NL}



Parameter estimation

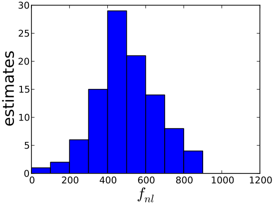
- We have 3 unknown parameters to constrain:

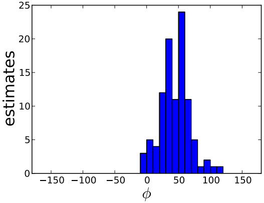
  amplitude f_{NL}

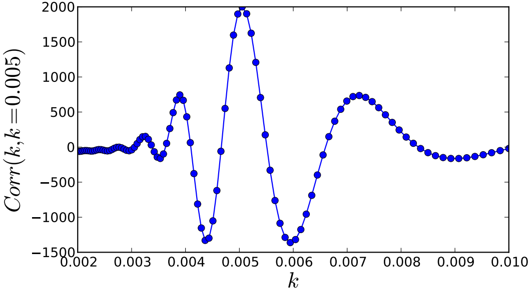
  phase ϕ

  frequency k_c

- Verification of the estimator via MC



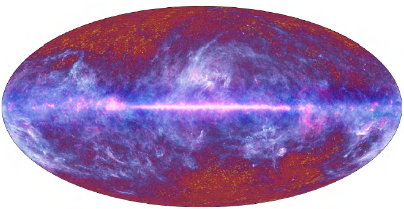


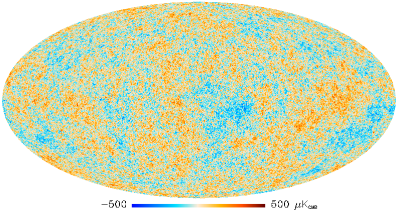


Real world estimator problems

Real data makes the analysis more complicated. Important systematics that have to be considered:

- Non-uniform and incomplete sky coverage
- Non-primordial contributions to the bispectrum (e.g. ISW-lensing)
- Foreground cleaning of the CMB maps
- Noise properties of the detector





Conclusions

- Non-gaussianities are a unique window into the physics of the early universe
- Despite so far negative results there is still room for improvement, also with CMB data
- Oscillating bispectra have not been extensively searched for and are theoretically well motivated
- The method developed here will be applied to the Planck data