Simulating the reionization of the Universe: *faster*, and *robuster*

Yi Mao
IAP/ILP

The first "ILP Day"
Paris, March 13, 2014

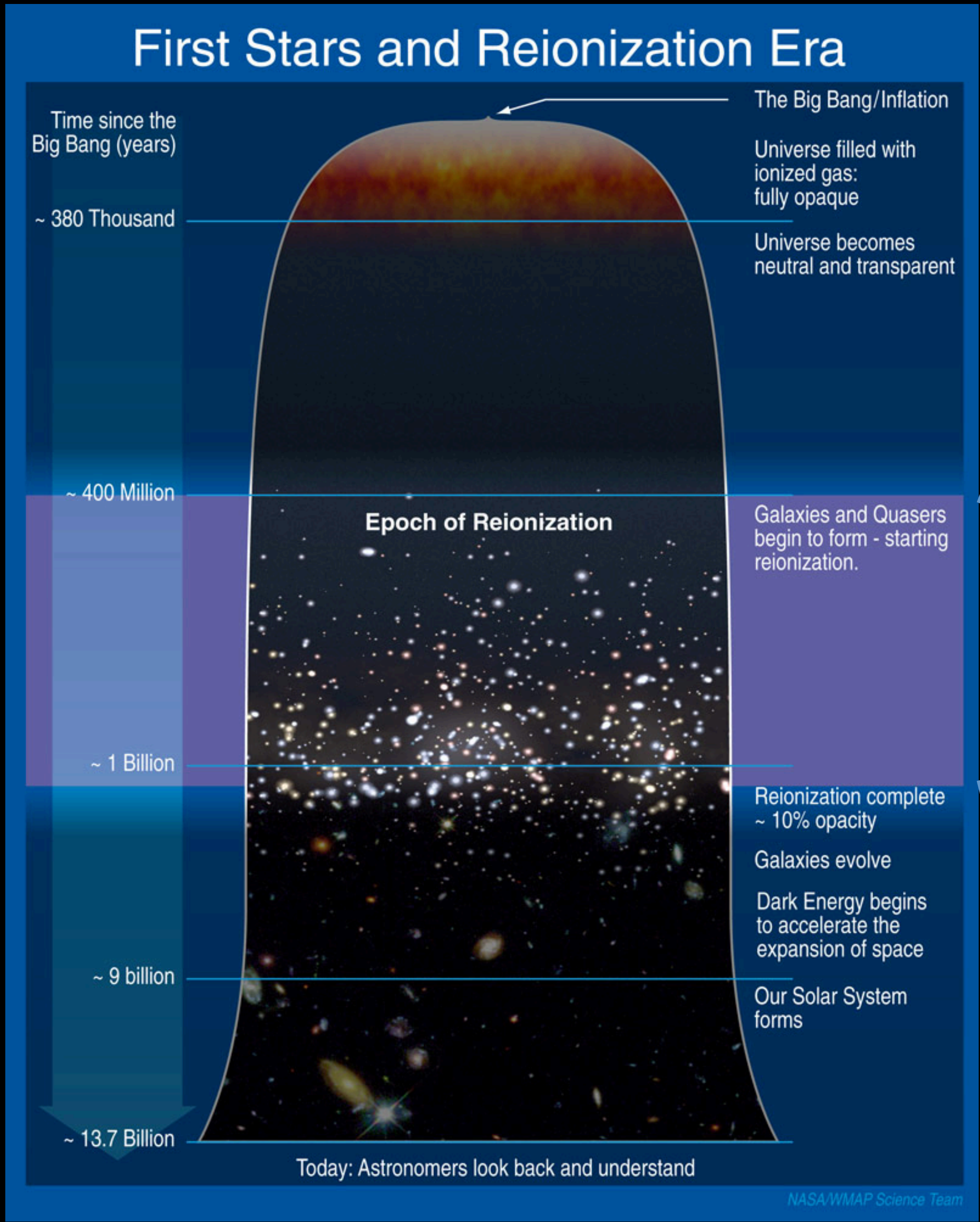
Collaborators: Ben Wandelt (IAP), Joe Silk (IAP), Benoit Semelin (Paris Obs.), Paul Shapiro (Texas), Jun Zhang (Shanghai Jiao Tong U.), Ilian Iliev (Sussex)

Simulating the reionization of the Universe: *faster*, and *robuster*

Yi Mao
IAP/ILP

The first "ILP Day"
Paris, March 13, 2014

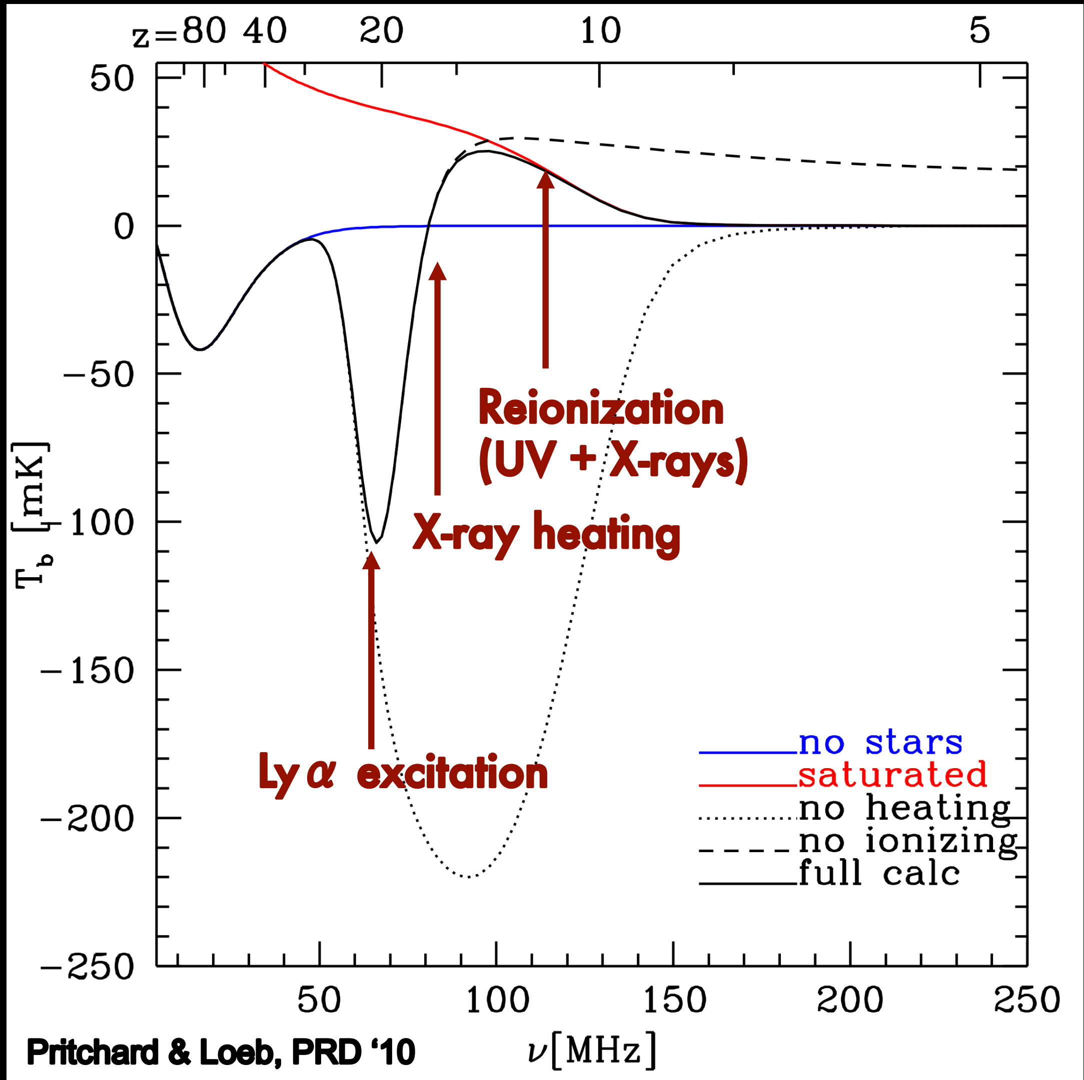
** I'd like to thank the Lagrange Institute for the support of my research through the Lagrange Fellowship, although I confess that I am still using the Eulerian grid in my simulation instead of the Lagrangian scheme.*



Credit: NASA/WMAP Science Team

Credit: A. Loeb '06

Observable: the 21-cm emission line



LOFAR, MWA, PAPER, 21CMA, GMRT, EDGES

SKA

LEDA, DARE

Approaches for reionization

✧ Analytical: excursion set model of reionization (a.k.a. bubble model)

Furlanetto+ '04

Consistent with intuitive picture and good for qualitative interpretation.

✧ Semi-analytical simulation: ionization criterion based on bubble model

21cmFAST (Mesinger+ '07, '11)

Choudhury+ '09

SimFAST21 (Santos+ '10)

Zahn+ '11

Fialkov+ '13

Fast, (really fast), but not so robust

✧ Numerical simulation: solve the radiative transfer equation of ionizing photons

Robust (with reasonable approximations), but computationally expensive

Numerical RT algorithms

✧ Ray-tracing methods

Abel+ '99, Abel & Wandelt '02
FLASH (Fryxell+ '00)
Sokasian+ '01
Razoumov+ '02, Razoumov & Cardall '05
C^2Ray (Mellema+ '06)
Susa '06
McQuinn+ '07
Trac & Cen '07
TRAPHIC (Pawlik & Schaye '08, '10)
START (Hasegawa & Umemura '10)
SimpleX, SimpleX2 (Kruip+ '10, Paardekooper+ '10)

Computational cost may be increasing with the number of sources. Accuracy may be reduced for long mean free path rays.

✧ Monte Carlo methods

CRASH, CRASH2 (Ciardi+ '01, Maselli+ '03, '09)
LICORICE (Semelin+ '07, Baek+ '09, '10)
SPHRAY (Altay+ '08)

Memory loads may increase dramatically for long mean free path photon packets.

✧ Moments methods

Gnedin & Abel '01
Whalen & Norman '06
ATON (Aubert & Teyssier '08, '10)
Finlator+ '09
AREPO (Petkova & Springel '08, '10, '11)

Edington tensor calculation may be tricky.

To be fast, or to be robust, that is the question.

Is the speed so dear as to be purchased at the price of robustness?

To be fast, or to be robust, that is the question.

Is the speed so dear as to be purchased at the price of robustness?

I'd rather not to say yes. And I want both, so...

Introducing the new algorithm for numerical RT simulation

F^2Ray

F^{2}Ray Idea 1: radiative transfer equation

$$\frac{a\partial I_\nu}{c\partial t} + \hat{n}\cdot\nabla I_\nu - Ha\frac{\partial I_\nu}{c\partial \ln\nu} = \frac{1}{4\pi}S - I_\nu \underbrace{\sum_{\mathrm{HI,HeI,HeII}} n_i \frac{\sigma_{\gamma i}(\nu)}{a^2}}_{\Gamma_\gamma(\mathbf{x},t,\nu)}$$

Photoionization rate

Advection term
Non-local in x-space
Local in k-space

$$\hat{n}\cdot\vec{k}\,\widetilde{I_\nu}(t,\mathbf{k},\nu,\hat{n})$$

Cosmological
redshifting

Source
emissivity
term

Photoionization term
Local in x-space
Convolution in k-space

F²Ray Idea 1: radiative transfer equation

$$\underline{\delta I_\nu}(t,\nu,\hat{n}) = \begin{bmatrix} \widetilde{\delta I_\nu}(\mathbf{k_1},t,\nu,\hat{n}) \\ \vdots \\ \widetilde{\delta I_\nu}(\mathbf{k_{N-1}},t,\nu,\hat{n}) \end{bmatrix}$$

$$\underline{\delta S}(t,\nu) = \begin{bmatrix} \widetilde{\delta S}(\mathbf{k_1},t,\nu) \\ \vdots \\ \widetilde{\delta S}(\mathbf{k_{N-1}},t,\nu) \end{bmatrix}$$

$$\underline{\delta \Gamma_\gamma}(t,\nu) = \begin{bmatrix} \widetilde{\delta \Gamma_\gamma}(\mathbf{k_1},t,\nu) \\ \vdots \\ \widetilde{\delta \Gamma_\gamma}(\mathbf{k_{N-1}},t,\nu) \end{bmatrix}$$

F^2Ray Idea 1: radiative transfer equation in Fourier space

$$\frac{\partial}{\partial v}\underline{\delta I_\nu} = \frac{c}{4\pi H a}\underline{\delta S} - \hat{P}\,\underline{\delta I_\nu}$$

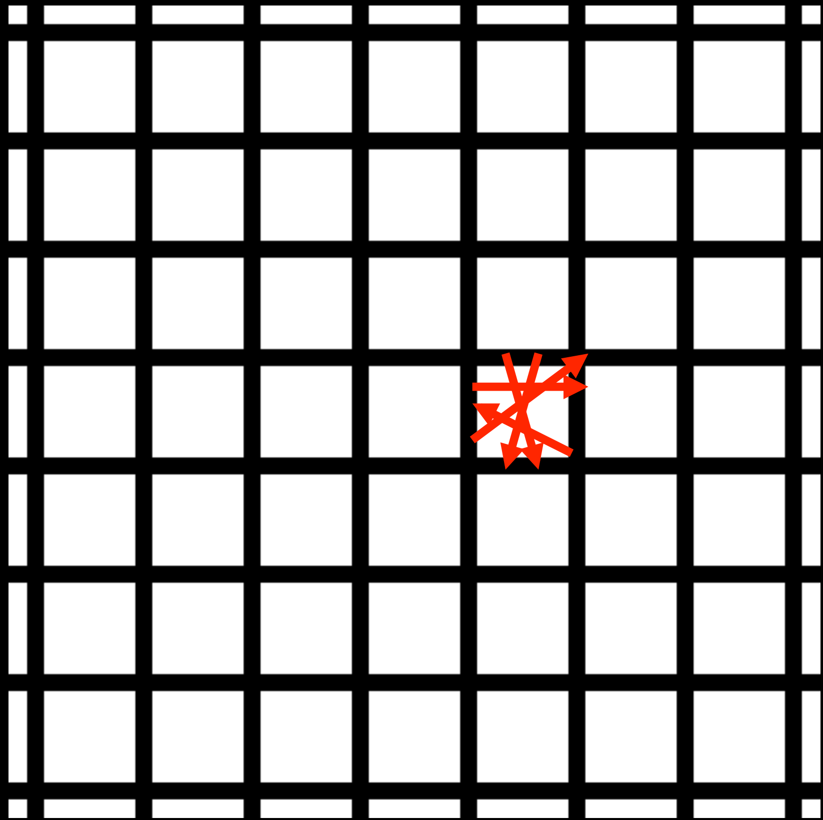
v is some time variable

$\hat{P}(t,\nu,\hat{n})$ **is a set of non-commutative matrices, i.e.**
$[\hat{P}(t_1,\nu,\hat{n}),\hat{P}(t_2,\nu,\hat{n})] \neq 0$

The matrix P contains the advection term (diagonal) and photoionization term (non-diagonal).

We find a formal solution to this equation, fortunately, using the time-ordering technique (developed in solving the time-varying Schrödinger equation, and in writing the Green function in quantum field theory).

F^{2}Ray Idea 2: photoionization



The gas doesn't care which direction the photons come from in terms of photoionization. Only the total flux (integrated over directions) matters!

$$\Gamma_{\gamma\,\mathrm{HI}} = c\int_{\nu_{\mathrm{th}}}^{\infty}\frac{d\nu}{\nu}\left[\int d^2\hat{n}\,I_\nu(t,\mathbf{x},\nu,\hat{n})\right]\frac{\sigma_{\gamma\,\mathrm{HI}}(\nu)}{a^2}\kappa(\nu,x_{\mathrm{HII}})$$

F^2Ray: Solution to RT equation in Fourier space

$$\int d^2\hat{n}\, \underline{I_\nu}(t,\nu,\hat{n})$$

$$= \int_0^t \frac{cdt'}{a'}\, \hat{R}(t,\nu,t')\underline{Q}(t',\nu')$$

Integration over past lightcone

"Radiative transfer matrix" from t' to t, which can be computed analytically.

Effective source at t' with ν'

F^2Ray = Fast Fourier Ray-tracing method

F^2Ray: pros

✧ The F^2Ray algorithm is based on analytic solution of RT equation in Fourier space. Robuster than ever!

✧ Computational cost is independent of the number of ionizing sources. Good for both point source and diffuse source.

✧ Good scaling law; in practice, ≈ 100 N ln(N), where N = the grid number. Faster than ever!

✧ No more memory for individual rays.

✧ Photon conserving.

✧ Automatically observe the periodical boundary condition.

✧ Can be used to both UV photons (short mean free path) and X-rays (long mean free path). Can be coupled to thermal equation, to simulate the thermal evolution of IGM, in addition to the reionization.

F^2Ray: cons

- Cannot be applied for ionizing source with anisotropic emissivity.
- Cannot be applied for open boundary condition.